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DESIGN AND ANALYSIS OF ORTHOTROPIC RING-STIFFENED CYLINDRICAL SHELLS SUBJECTED TO EXTERNAL HYDROSTATIC PRESSURE

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Three theories are presented for the analysis of ring-stiffened or monocoque cylindrical shells under hydrostatic pressure. The theories compute the stress state and the axisymmetric collapse, interbay buckling, and general instability pressures from external hydrostatic pressure loading. The theories were derived for the case of specially orthotropic material with theoretically uniform properties through the thickness; the theories apply equally to shells made of isotropic materials. A computer program, DAPS4 (Design and Analysis of Plastic Shells, Version 4), was written that incorporates these theories that can be used as an initial sizing tool or analysis of a shell's hydrostatic pressure capability. The program includes a "pseudo-plastic" method to account for the reduction of computed collapse pressures when the stress state is above the proportional limit of the material's stress-strain curve. The purpose of this report is to document the theories and the code. Correlation with numerous pressure tests is shown. A user manual with example problems is included.

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FOREWORD

Three theories are presented for the analysis of ring-stiffened or monocoque cylindrical shells under hydrostatic pressure. The theories compute the stress state and the axisymmetric collapse, interbay buckling, and general instability pressures from external hydrostatic pressure loading. The theories were derived for the case of specially orthotropic material with theoretically uniform properties through the thickness; the theories apply equally to shells made of isotropic materials. A computer program, DAPS4 (Design and Analysis of Plastic Shells, Version 4), was written that incorporates these theories that can be used as an initial sizing tool or analysis of a shell's hydrostatic pressure capability. The program includes a "pseudo-plastic" method to account for the reduction of computed collapse pressures when the stress state is above the proportional limit of the material's stress-strain curve. The purpose of this report is to document the theories and the code. Correlation with numerous pressure tests is shown. A user manual with example problems is included.

The theories reported herein were developed as part of the author's independent research in the area of shell buckling over the past 22 years offering a number of improvements to the prior theories and the DAPS3 computer program. Navy projects that have benefited over the years from the application of the DAPS3 and DAPS4 codes include the CVLWT, Long Pulse, Metal Matrix Composites, MK 46 Torpedo, and MK 48 Torpedo.

Amos Dare
Manager, Explosives Effects Branch

Approved and released by:

Jonathan Ross
Head, Energetics Technology Department

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DAPS4 THEORY AND EQUATIONS

DAPS4 is an inexpensive and powerful design computer program for designing or analyzing cylindrical shells (ring stiffened or monocoque) subjected to external hydrostatic pressure. The material may be isotropic or specially orthotropic or a hybrid of both (e.g., the rings may be one material and the shell plating another). Like the DAPS3 version of the code documented in reference 1, the DAPS4 code computes the stresses and deflections, interbay buckling pressure, general instability pressure, and the axisymmetric collapse pressure of a ring-stiffened cylindrical shell. In addition to analysis, both codes will design a ring-stiffened or monocoque shell based on weight minimization. A “pseudo-plastic” method of analysis is used which uses the hoop, axial, and ring tangent moduli which are determined by the corresponding hoop, axial, and ring stresses in the shell and the material stress-strain curves. The application supports mines, torpedoes, submarines, or materials research. The relevant leadership areas are undersea vehicle hulls, mines, warheads, and special warfare systems.

Interbay buckling is a non-symmetric instability that is localized between two stiffeners. Inward and outward lobes are formed alternately around the circumference (in accordance with the number of circumferential waves n appearing in the equations in the following sections). General instability is a non-symmetric collapse of the full length of the shell, i.e., buckling of the rings and shell plating together. Axisymmetric collapse is a combined yielding and buckling phenomenon that is precipitated by axisymmetric yielding in the shell plating between two rings (a bay); collapse occurs when three plastic hinges form: at mid-bay and at the two rings bordering that bay.

The new theories and options in DAPS4 include the following that are not in DAPS3:

a. A new interbay buckling theory that accounts for the ring’s stiffness in torsion and in plane and out-of-plane bending, eliminating the simple support assumption at the bay ends.

b. Stresses and deflections at all points between the ring and mid-bay.

c. Added ring stiffener options include reading in ring “parts” for ease of analysis of general Tee-stiffeners, including a faying flange; rings may be internal or external.

d. A theory is incorporated to take advantage of the plastic reserve strength in the shell in the formation of three fully plastic hinges in the computation of the axisymmetric collapse mode (applicable to isotropic materials only). (DAPS3 used yielding at the outer surface at mid-bay as the criterion to indicate the onset of axisymmetric collapse.)

Interbay Buckling

An example of interbay buckling, a non-symmetric instability that is localized between the stiffeners, is shown in Figure 1. The assumed buckled shape for interbay buckling used in DAPS3 inherently assumes, conservatively, simple support at the rings and takes the form

$$u = A\sin n\phi \sin\frac{\alpha x}{r}$$
$$v = B\cos n\phi \cos\frac{\alpha x}{r} \qquad (1)$$
$$w = \sin n\phi \cos\frac{\alpha x}{r}$$

where $\alpha = \dfrac{\pi r}{L_s}$ and $x = 0$ at mid-bay for equations (1), as indicated in Figure 2.

Figure 1. Non-Symmetric Local Buckling

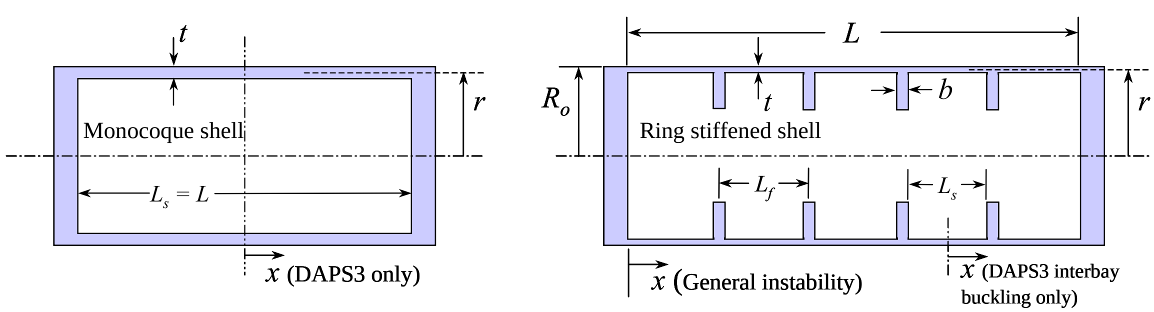


Figure 2. Monocoque and Internally Stiffened Cylindrical Shell

For the DAPS4 code, the ring's torsion and bending stiffness influences the interbay buckling pressure calculation (to remove the conservatism inherent in the DAPS3 solution) and is accounted for by assuming a series expansion in the buckled shape which takes the form

$$
\begin{aligned}
&u = c_n \sum_{i=1}^{K} c_i U_i \\
&v = s_n \sum_{j=1}^{K} s_j V_j \\
&w = c_n \sum_{k=1}^{K} s_k W_k \\
&c_n = \cos n\phi \,,\; s_n = \sin n\phi \\
&c_i = \cos \frac{m_i \pi x}{L} \,,\; m_i = n_i (N+1) \,,\; n_i = [2(i-1)+1] \\
&s_j = \sin \frac{m_j \pi x}{L} \,,\; m_j = n_j (N+1) \,,\; n_j = [2(j-1)+1] \\
&s_k = \sin \frac{m_k \pi x}{L} \,,\; m_k = n_k (N+1) \,,\; n_k = [2(k-1)+1]
\end{aligned}
\tag{2}
$$

where $x = 0$ at one end of the ring-stiffened shell, as indicated in Figure 3.

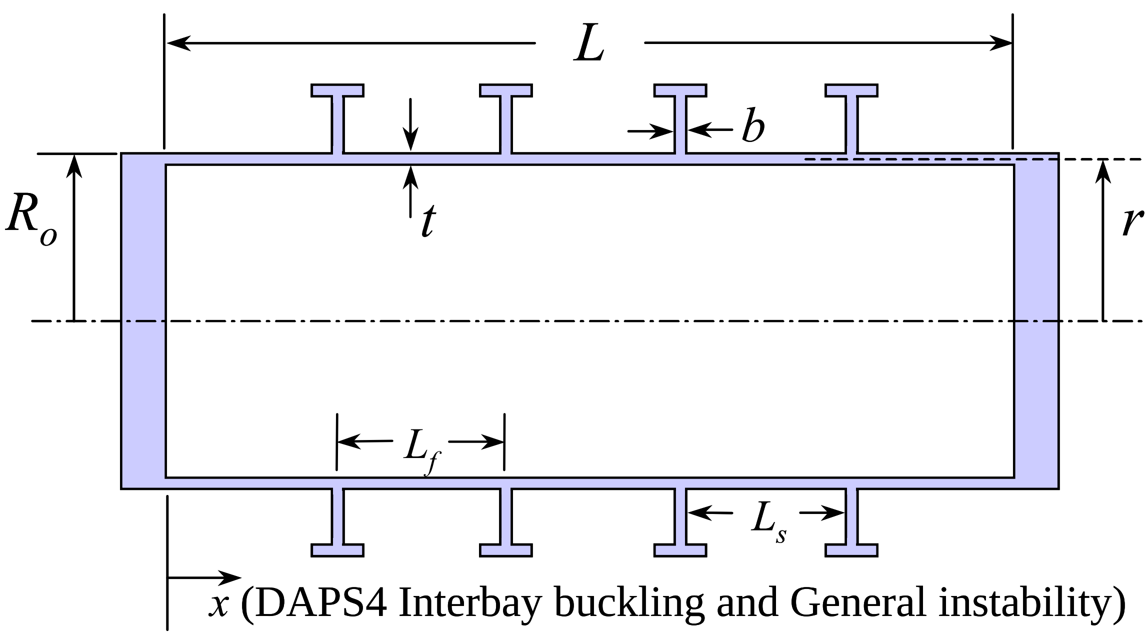


Figure 3. Externally Tee-Stiffened Cylindrical Shell

This expression was chosen because it satisfies the boundary conditions necessary to develop the realistic influence on the buckled shape due to the torsion and out-of-plane bending of the rings. That is, the m_i, m_j, and m_k are always an odd number of half waves per bay, requiring the slope of the buckled shape at any ring to be

negative the slope at an adjacent ring. For example, note that for the s_k term with k = 1, n_k is equal to 1 and m_k = N + 1 which makes s_k one-half sine wave per bay since N + 1 is the number of bays (with N = the number of stiffening rings). When the series is summed over i, j, k, the energy minimization technique will compute an eigenvector comprised of the U_i, V_j, and W_k coefficients that scale, positive or negative, the individual terms that serve to contribute to the bifurcation buckling shape corresponding to the minimum total strain and work energy.

For the DAPS3 solution for the interbay buckling pressure, three equilibrium equations were developed (and documented in reference 1) into which equations (1) were substituted, leading to a solution for the buckling pressure as a function of the number of hoop waves; the critical buckling pressure is the minimum of these values.

The DAPS4 solution for the interbay buckling pressure is found by using the energy method of Ritz in a manner similar to the method for obtaining the general instability pressure. The procedure is summarized as follows:

a. The displacements are segregated into their pre-buckling and post-buckling parts in accordance with
$$u = u_0 + u_1 \,,\; v = v_0 + v_1 \,,\; w = w_0 + w_1 \,. \tag{3}$$

b. Work is done by external pressure:
$$W_p = \frac{-\mathrm{Pr}}{2}\int_0^{2\pi}\int_0^{L}(2wu_x + \frac{2wv_\phi}{r} - v_\phi u_x - \frac{w^2}{r})d\phi dx \,. \tag{4}$$

c. Strain energy is stored in the rings and shell plating.

d. The pre-buckled stress state is derived using the stress equations that were originally derived in reference 2 and re-written in a more general format in reference 1.

e. The strain energy is minimized with respect to the assumed buckled shape given in equations (2).

f. The critical buckling pressure, P_{cr}, is the minimum positive eigenvalue found in the solution of the equations.

In DAPS3 (for general instability) and DAPS4 (for interbay and general instability), the strain energy in the shell plating is comprised of two parts, U = U_e and U_b:

U_e = shell extensional strain energy.

$$\begin{aligned} U_e = {} & \frac{r}{2}\int_0^{2\pi}\int_0^{L}[C_x {u_x}^2 + \frac{C_\phi}{r^2}(v_\phi - w)^2 + 2\upsilon_{\phi x} C_x \frac{u_x}{r}(v_\phi - w)]d\phi dx \\ & + \frac{G_{LT}tr}{2}\int_0^{2\pi}\int_0^{L}(\frac{u_\phi}{r} + v_x)^2 d\phi dx \\ & - \frac{\mathrm{Pr}^2\,\alpha^2}{4}\int_0^{2\pi}\int_0^{L}({v_x}^2 + {w_x}^2)d\phi dx \\ & + \frac{P(\gamma_s^e - r)}{2r}\int_0^{2\pi}\int_0^{L}({u_\phi}^2 + {w_\phi}^2 - 2wv_\phi)d\phi dx \end{aligned} \tag{5}$$

where

$$\gamma_s^e = \frac{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})\beta}{R\left(\frac{K'}{F_1'}+\frac{4\theta^4 D_x}{L_s^{\ 3}}\right)} \tag{6}$$

and

$$\beta = \frac{E_f A_{eff}(1-\frac{\upsilon_{\phi x}\alpha^2}{2})}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})} + \frac{b\alpha}{2}(\upsilon_{x\phi}-\upsilon_{\phi x}\alpha)$$

$$K' = \frac{1}{R^2}[E_f A_{eff} + bC_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})]$$

$b =$ the width of the ring where it contacts the shell.

$$F_1' = \frac{2}{\theta}\left[\frac{\cosh\theta-\cos\theta}{\sinh\theta+\sin\theta}\right] \tag{7}$$

$$\theta = \sqrt[4]{\frac{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}{4D_x R^2}}L_s$$

$$A_{eff} = A_f\left(\frac{R}{R_{cg}}\right)^2 = \text{"effective ring area"};\ A_f = \text{ring area.}$$

$$\alpha = R_o / R$$

$$R \equiv r \ \text{(used interchangeably).}$$

The remaining geometry variables are defined in Figure 3.

U_b = shell bending and twisting strain energy.

$$U_b = \frac{1}{2r}\int_0^{2\pi}\int_0^L [D_x r^2 w_{xx}^{\ 2} + \frac{D_\phi}{r^2}(w_{\phi\phi}+w)^2 + 2\upsilon_{\phi x}D_x w_{xx}(w_{\phi\phi}+w)]d\phi dx$$
$$+\frac{G_{LT}t^3}{6r}\int_0^{2\pi}\int_0^L (w_{x\phi}+\frac{v_x}{2}-\frac{u_\phi}{2r})^2 d\phi dx\,. \tag{8}$$

The subscripts x and ϕ of the displacements u, v, and w in equations (5) and (8) are partial derivatives in the axial and hoop directions, respectively, of the assumed displacement form given in equations (2).

The strain energy in the rings is comprised of two parts in DAPS3 (for general instability only) and three parts in DAPS4 (for general and interbay instability).

In DAPS3: $V = V_e + V_b$.

V_e = in-plane extension.

$$V_e = \frac{E_f A_f}{2r}\int_0^{2\pi}[(w_{\phi\phi}+w)\frac{e}{r}-(v_\phi - w)]^2 d\phi$$
$$+\frac{PE_f A_f \gamma_f^e}{t}\int_0^{2\pi}[(w_{\phi\phi}+w)\frac{e}{r}-(v_\phi - w)]v_\phi d\phi \tag{9}$$
$$-\frac{PE_f A_f \gamma_f^e (1+\frac{e}{r})}{2t}\int_0^{2\pi}(u_\phi{}^2 + w_\phi{}^2 - 2wv_\phi)d\phi$$

where

$$\gamma_f^e = \frac{-t}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}\left\{-(1-\frac{\upsilon_{\phi x}\alpha^2}{2})+\beta\Big/\left[\frac{E_f A_{eff}}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}+b+L_s F_1'\right]\right\}. \tag{10}$$

V_b = in-plane bending.

$$V_b = \frac{E_f I_r}{2r^3}\int_0^{2\pi}(w_{\phi\phi}+w)^2 d\phi \tag{11}$$

where I_r is the moment of inertia of a ring about a line through its centroid and parallel to the cylinder's axis.

The radial eccentricity, e, of the ring centroid to the shell mid-surface is positive for internal rings in the derivation of equation (9). Equations (9) and (11) are summed over all of the discrete rings whose axial positions are at $x = aL_f, \sum_{a=1}^{N}$, where L_f is the ring spacing.

In DAPS4: $V = V_e + V_b + V_r^{(a)}$, where V_e and V_b are given by equations (9) and (11), respectively.

$V_r^{(a)}$ = additional ring strain energy from out-of-plane bending and torsion.

$$\begin{aligned} V_r^{(a)} = {} & \frac{E_f I_x}{2r_c^3}\int_0^{2\pi}\begin{pmatrix} u_{\phi\phi}{}^2 + e_1{}^2 w_{x\phi\phi}{}^2 + r_c{}^2 w_x{}^2 + 2e_1 u_{\phi\phi}w_{x\phi\phi} \\ + 2r_c u_{\phi\phi}w_x + 2e_1 r_c w_x w_{x\phi\phi}\end{pmatrix} d\phi \\ & - \frac{E_f I_{xy}}{rr_c^2}\int_0^{2\pi}\begin{pmatrix} u_{\phi\phi}w_{\phi\phi} + e_1 w_{\phi\phi}w_{x\phi\phi} + r_c w_x w_{\phi\phi} + u_{\phi\phi}v_\phi \\ + e_1 v_\phi w_{x\phi\phi} + r_c v_\phi w_x \end{pmatrix} d\phi \quad , x = aL_f, \sum_{a=1}^{N}. \\ & + \frac{G_f J}{2r_c^3}\int_0^{2\pi}(r^2 w_{x\phi} - 2ru_\phi w_{x\phi} + u_\phi{}^2)d\phi \end{aligned} \tag{12}$$

Equation (12) is summed over all of the discrete rings whose axial positions are at $x = aL_f, \sum_{a=1}^{N}$, as was done with equations (9) and (11). Also, the subscripts x and ϕ of the displacements u, v, and w in equations (9), (11), and (12) are partial derivatives in the axial and hoop directions, respectively, of the assumed displacement form given in equations (2). The radial eccentricity, e_1, of the ring centroid to the shell mid-surface is positive for external rings in the derivation of equation (12).

Elaboration on Equation (12)

Equations (4) through (11) were derived (including the intermediate strain-displacement relationships necessary to represent the strain energy in terms of the displacement components and their derivatives) and fully documented in reference 1 in their expanded form. Since the derivation and expansion of equation (12) was (until now) undocumented, this will be provided here.

To begin, reference is made to the strain-displacement and curvature-displacement relationships used by Bushnell in the BOSOR4 code (reference 3):

$$\{e\} = \begin{Bmatrix} e_1 \\ e_2 \\ e_{12} \\ \kappa_1 \\ \kappa_2 \\ 2\kappa_{12} \end{Bmatrix} = \begin{Bmatrix} u' + w/R_1 + \tfrac{1}{2}(\beta^2 + \gamma^2) \\ \dot{v}/r + ur'/r + w/R_2 + \tfrac{1}{2}(\psi^2 + \gamma^2) \\ \dot{u}/r + r(v/r)' + \beta\psi \\ \beta' \\ \dot{\psi}/r + r'\beta/r \\ 2(-\dot{\beta}/r + r'\psi/r + v'/R_2) \end{Bmatrix} \tag{13}$$

in which

$$\beta = w' - u/R_1$$

$$\psi = \dot{w}/r - v/R_2$$

$$\gamma = \tfrac{1}{2}(\dot{u}/r - v' - r'v/r)$$

and

' indicates d/ds, i.e., the derivative with respect to the meridional direction s.

$\cdot$ indicates $d/d\theta$, i.e., the derivative with respect to the circumferential direction θ

and R_1 and R_2 are the meridianal and normal circumferential radii of curvature. These expressions for $\{e\}$ are used in BOSOR4 and are valid for moderately large relations. For the special case of right circular cylindrical shells analyzed in DAPS4, these relations reduce, recognizing that $R_1 = \infty$, BOSOR4 s coordinate $\equiv$ DAPS4 x coordinate, BOSOR4 θ coordinate $\equiv$ - DAPS4 ϕ coordinate, $R_2 = r$, and $dr/ds = 0$.

From reference 3, the discrete ring strain energy for use in BOSOR4 is

$$U_r = \frac{r_c}{2}\int_\theta\int_A \sigma_r(\varepsilon_r - \alpha_r T)\,dA + \frac{G_f J}{2r_c}\int_\theta (\dot{\beta} + \dot{u}_c/r_c)^2\,d\theta \tag{14}$$

where A is the ring cross sectional area, G_fJ is the ring torsional rigidity, and the subscript c refers to the ring centroid. $\alpha_r T$ is a temperature effect ignored in DAPS4. The ring hoop stress is given by

$$\sigma_r = E_f\varepsilon_r = E_f(e_r - x\kappa_x + y\kappa_y) \tag{15}$$

where

$$
\begin{aligned}
&e_r = \frac{\dot{v}_c}{r_c} + \frac{w_c}{r_c} + \frac{1}{2}(\psi_c{}^2 + \gamma_c{}^2) \\
&\kappa_x = \frac{\dot{\psi}_c}{r_c}\,,\ \kappa_y = -\frac{\dot{\gamma}_c}{r_c} + \frac{\beta}{r_c} \\
&\psi_c = (\dot{w}_c - v_c)/r_c\,,\ \gamma_c = \dot{u}_c/r_c \\
&\text{and } \beta = w' \text{ since } R_1 = \infty.
\end{aligned}
\tag{16}
$$

When equation (15) is substituted into (14), the temperature effect dropped, and the first term integrated over the ring cross sectional area, equation (14) becomes

$$
\begin{aligned}
U_r = {}& \frac{r_c}{2}\int_\theta (E_f A_f e_r{}^2 + E_f \kappa_x{}^2 I_y + E_f \kappa_y{}^2 I_x - 2\kappa_x \kappa_y E_f I_{xy})\,d\theta \\
&+ \frac{G_f J}{2r_c}\int_\theta (\dot{\beta} + \dot{u}_c/r_c)^2\,d\theta
\end{aligned}
\tag{17}
$$

where I_x, I_y, and I_{xy} are the moments and product of inertia about the ring's cross section centroidal axes, x and y. (In the ring cross-section context, x is the radial direction and y is parallel to the cylinder axis.)

Dropping the terms for strain energy already taken into account by equations (9) and (11), i.e., for in-plane extension and in-plane bending, respectively, the terms remaining are the components of the ring strain energy (out-of-plane bending and torsion) that need to be added. Thus, the "added ring strain energy", $V_r^{(a)}$, that was not needed for DAPS3 but is needed for DAPS4 is

$$
\begin{aligned}
V_r^{(a)} = {}& \frac{E_f r_c}{2}\int_\theta (\kappa_y{}^2 I_x - 2\kappa_x \kappa_y I_{xy})\,d\theta \\
&+ \frac{G_f J}{2r_c}\int_\theta (\dot{\beta} + \dot{u}_c/r_c)^2\,d\theta.
\end{aligned}
\tag{18}
$$

For the cylindrical shell, the ring displacements (at its centroid) u_c, v_c, and w_c coincide with the shell u, v, and w directions, respectively, but not in magnitude. The radial eccentricity of the ring centroid to the shell mid-surface is e_1, positive for external rings in the derivation of $V_r^{(a)}$ only. The displacements are defined as:

$$
u_c = u + \Delta u^*,\ v_c = v + \Delta v^*,\ w_c = w + \Delta w^*. \tag{19}
$$

Using the definitions for $\Delta u^*, \Delta v^*,$ and Δw^* from reference 3, setting the axial eccentricity of the ring-to-shell location to zero (i.e., $e_2 = 0.$), noting that $R_1 = \infty$ for the cylindrical case, and rewriting in the DAPS4 coordinate system[1], equations (19) can be written as

[1]In DAPS4, positive w displacement is radially inward in the buckling derivations (reverse of BOSOR4), which makes the DAPS4 displacement v and circumferential coordinate ϕ the reverse of the BOSOR4 displacement v and circumferential coordinate θ, respectively. This affects the signs of the individual terms in the definitions of the curvature-displacement relationships, κ_x and κ_y. The lengthy re-derivation of these terms is not provided here.

$$
\begin{aligned}
u_c &= u + e_1 w_x \\
v_c &= v + \frac{e_1}{r}(v + w_\phi - u_\phi w_x) \\
w_c &= w + e_1 \frac{{w_x}^2}{2}.
\end{aligned}
\tag{20}
$$

Also, the expression for ε_r, κ_x, and κ_y in equations (15) and (16) are rewritten in terms of the DAPS4 coordinate system as

$$
\begin{aligned}
\varepsilon_r &= e_r + x\kappa_x - y\kappa_y \\
\kappa_x &= \frac{1}{{r_c}^2}(w_{c,\phi\phi} + v_{c,\phi}) \\
\kappa_y &= \frac{1}{r_c}\left(\frac{u_{c,\phi\phi}}{r_c} + w_x\right).
\end{aligned}
\tag{21}
$$

Equation (18) is also re-derived in the DAPS4 coordinate system to yield the following equation:

$$
\begin{aligned}
V_r^{(a)} &= \frac{E_f r_c}{2}\int_0^{2\pi}({\kappa_y}^2 I_x - 2\kappa_x\kappa_y I_{xy})d\phi \\
&+ \frac{G_f J}{2r_c}\int_0^{2\pi}(w_{x\phi} - \frac{u_{c,\phi}}{r_c})^2 d\phi.
\end{aligned}
\tag{22}
$$

Substituting equations (20) and (21) into (22), and carrying out all the necessary differentiations, results in the following equation for the added ring strain energy due to torsion and out-of-plane bending of the ring cross section:

$$
\begin{aligned}
V_r^{(a)} &= \frac{E_f I_x}{2{r_c}^3}\int_0^{2\pi}\begin{matrix}({u_{\phi\phi}}^2 + {e_1}^2 {w_{x\phi\phi}}^2 + {r_c}^2 {w_x}^2 + 2e_1 u_{\phi\phi} w_{x\phi\phi} \\ + 2r_c u_{\phi\phi} w_x + 2e_1 r_c w_x w_{x\phi\phi})d\phi\end{matrix} \\
&- \frac{E_f I_{xy}}{r{r_c}^2}\int_0^{2\pi}\begin{matrix}(u_{\phi\phi} w_{\phi\phi} + e_1 w_{\phi\phi} w_{x\phi\phi} + r_c w_x w_{\phi\phi} + u_{\phi\phi} v_\phi \\ + e_1 v_\phi w_{x\phi\phi} + r_c v_\phi w_x)d\phi\end{matrix} \quad , x = aL_f, \sum_{a=1}^{N} \\
&+ \frac{G_f J}{2{r_c}^3}\int_0^{2\pi}(r^2 w_{x\phi} - 2ru_\phi w_{x\phi} + {u_\phi}^2)d\phi
\end{aligned}
\tag{23}
$$

which is what was given for equation (12). The subscripts x and ϕ of the displacements u, v, and w in equation (23) are partial derivatives in the axial and hoop directions, respectively, of the assumed displacement form given in equations (2).

Total Energy

In accordance with equations (4) through (12), the total strain and work energy accounted for in DAPS4 is:

$$
U_T = U_e + U_b + V_e + V_b + V_r^{(a)} + W_p \tag{24}
$$

or

$$U_T = \frac{r}{2}\int_0^{2\pi}\int_0^L [C_x u_x{}^2 + \frac{C_\phi}{r^2}(v_\phi - w)^2 + 2\upsilon_{\phi x} C_x \frac{u_x}{r}(v_\phi - w)] d\phi dx$$
$$+ \frac{G_{LT} tr}{2}\int_0^{2\pi}\int_0^L (\frac{u_\phi}{r} + v_x)^2 d\phi dx$$
$$- \frac{\Pr^2 \alpha^2}{4}\int_0^{2\pi}\int_0^L (v_x{}^2 + w_x{}^2) d\phi dx$$
$$+ \frac{P(\gamma_s^e - r)}{2r}\int_0^{2\pi}\int_0^L (u_\phi{}^2 + w_\phi{}^2 - 2wv_\phi) d\phi dx$$
$$+ \frac{1}{2r}\int_0^{2\pi}\int_0^L [D_x r^2 w_{xx}{}^2 + \frac{D_\phi}{r^2}(w_{\phi\phi} + w)^2 + 2\upsilon_{\phi x} D_x w_{xx}(w_{\phi\phi} + w)] d\phi dx$$
$$+ \frac{G_{LT} t^3}{6r}\int_0^{2\pi}\int_0^L (w_{x\phi} + \frac{v_x}{2} - \frac{u_\phi}{2r})^2 d\phi dx$$
$$+ \frac{E_f A_f}{2r}\int_0^{2\pi} [(w_{\phi\phi} + w)\frac{e}{r} - (v_\phi - w)]^2 d\phi,\ x = aL_f, \sum_{a=1}^{N}$$
$$+ \frac{P E_f A_f \gamma_f^e}{t}\int_0^{2\pi} [(w_{\phi\phi} + w)\frac{e}{r} - (v_\phi - w)] v_\phi d\phi,\ x = aL_f, \sum_{a=1}^{N}$$
$$- \frac{P E_f A_f \gamma_f^e (1 + \frac{e}{r})}{2t}\int_0^{2\pi} (u_\phi{}^2 + w_\phi{}^2 - 2wv_\phi) d\phi,\ x = aL_f, \sum_{a=1}^{N}$$
$$+ \frac{E_f I_r}{2r^3}\int_0^{2\pi} (w_{\phi\phi} + w)^2 d\phi,\ x = aL_f, \sum_{a=1}^{N}$$
$$+ \frac{E_f I_x}{2r_c{}^3}\int_0^{2\pi} \begin{matrix}(u_{\phi\phi}{}^2 + e_1{}^2 w_{x\phi\phi}{}^2 + r_c{}^2 w_x{}^2 + 2e_1 u_{\phi\phi} w_{x\phi\phi} \\ + 2r_c u_{\phi\phi} w_x + 2e_1 r_c w_x w_{x\phi\phi}) d\phi\end{matrix},\ x = aL_f, \sum_{a=1}^{N}$$
$$- \frac{E_f I_{xy}}{r r_c{}^2}\int_0^{2\pi} \begin{matrix}(u_{\phi\phi} w_{\phi\phi} + e_1 w_{\phi\phi} w_{x\phi\phi} + r_c w_x w_{\phi\phi} + u_{\phi\phi} v_\phi \\ + e_1 v_\phi w_{x\phi\phi} + r_c v_\phi w_x) d\phi\end{matrix},\ x = aL_f, \sum_{a=1}^{N}$$
$$+ \frac{G_f J}{2r_c{}^3}\int_0^{2\pi} (r^2 w_{x\phi} - 2ru_\phi w_{x\phi} + u_\phi{}^2) d\phi,\ x = aL_f, \sum_{a=1}^{N}$$
$$- \frac{\Pr}{2}\int_0^{2\pi}\int_0^L (2wu_x + \frac{2wv_\phi}{r} - v_\phi u_x - \frac{w^2}{r}) d\phi dx. \qquad (25)$$

Substituting equations (2) into (25) allows the total strain energy to be written as:

$$U_T = \begin{bmatrix} \sum_{i=1}^{K} A_{[3(i-1)+1],[3(i-1)+1]} U_i^{\,2} \\ + \sum_{j=1}^{K} A_{[3(j-1)+2],[3(j-1)+2]} V_j^{\,2} \\ + \sum_{k=1}^{K} A_{[3(k-1)+3],[3(k-1)+3]} W_k^{\,2} \end{bmatrix} \text{main diagonal terms only}$$

$$\begin{bmatrix} + 2\sum_{i=1}^{K}\sum_{j=1}^{K} A_{[3(i-1)+1],[3(j-1)+2]} U_i V_j \\ + 2\sum_{i=1}^{K}\sum_{k=1}^{K} A_{[3(i-1)+1],[3(k-1)+3]} U_i W_k \\ + 2\sum_{j=1}^{K}\sum_{k=1}^{K} A_{[3(j-1)+2],[3(k-1)+3]} V_j W_k \end{bmatrix} \text{terms above the diagonal of every 3x3 submatrix}$$

$$\begin{bmatrix} + 2\sum_{r=1}^{K}\sum_{s=1}^{K} A_{[3(r-1)+1],[3(s-1)+1]} U_r U_s \text{ , for } r \neq s \\ + 2\sum_{r=1}^{K}\sum_{s=1}^{K} A_{[3(r-1)+2],[3(s-1)+2]} V_r V_s \text{ , for } r \neq s \\ + 2\sum_{r=1}^{K}\sum_{s=1}^{K} A_{[3(r-1)+3],[3(s-1)+3]} W_r W_s \text{ , for } r \neq s \end{bmatrix} \text{diagonals of every 3x3 submatrix except main diagonal}$$

$$\begin{bmatrix} + 2\sum_{j=1}^{K}\sum_{i=1}^{K} A_{[3(j-1)+2],[3(i-1)+1]} U_i V_j \\ + 2\sum_{k=1}^{K}\sum_{i=1}^{K} A_{[3(k-1)+3],[3(i-1)+1]} U_i W_k \\ + 2\sum_{k=1}^{K}\sum_{j=1}^{K} A_{[3(k-1)+3],[3(j-1)+2]} V_j W_k \end{bmatrix} \text{terms below the diagonal of every 3x3 submatrix.} \tag{26}$$

The energy minimization technique requires setting the following partial derivatives to zero:

$$\frac{\partial U_T}{\partial U_i} = \frac{\partial U_T}{\partial V_i} = \frac{\partial U_T}{\partial W_i} = 0 \text{ , } i = 1, K \tag{27}$$

which leads to

$$2[\mathrm{A}]\{\mathrm{X}\} = \{0\} \tag{28}$$

where

$[\mathrm{A}] \equiv$ a square matrix of order 3K x 3K

$\{\mathrm{X}\} \equiv$ a vector of length 3K consisting of $U_i, V_i, W_i, i = 1, K$.

By way of example, partial differentiation of equation (26) with respect to U_i when K = 3 is

$$\begin{aligned}
\frac{\partial U_T}{\partial U_i} &= 2A_{11}U_1 + 2A_{44}U_2 + 2A_{77}U_3 \\
&+ 2A_{12}V_1 + 2A_{42}V_1 + 2A_{72}V_1 + 2A_{12}V_2 + 2A_{42}V_2 + 2A_{72}V_2 + 2A_{12}V_3 + 2A_{42}V_3 + 2A_{72}V_3 \\
&+ 2A_{12}W_1 + 2A_{42}W_1 + 2A_{72}W_1 + 2A_{12}W_2 + 2A_{42}W_2 + 2A_{72}W_2 + 2A_{12}W_3 + 2A_{42}W_3 + 2A_{72}W_3 \\
&+ 2A_{14}U_2 + 2A_{17}U_3 + 2A_{41}U_1 + 2A_{47}U_3 + 2A_{71}U_1 + 2A_{74}U_2 \\
&+ 2A_{21}V_1 + 2A_{24}V_1 + 2A_{27}V_1 + 2A_{21}V_2 + 2A_{24}V_2 + 2A_{27}V_2 + 2A_{21}V_3 + 2A_{24}V_3 + 2A_{27}V_3 \\
&+ 2A_{21}W_1 + 2A_{24}W_1 + 2A_{27}W_1 + 2A_{21}W_2 + 2A_{24}W_2 + 2A_{27}W_2 + 2A_{21}W_3 + 2A_{24}W_3 + 2A_{27}W_3.
\end{aligned} \tag{29}$$

With the 2 factored out in equation (28) and the individual terms in the square matrix $[\mathrm{A}]$ segregated into terms including the pressure, P, and those not involving P, equation (28) can be rewritten as

$$[a]\{\mathrm{X}\} - P[b]\{\mathrm{X}\} = \{0\} \tag{30}$$

where there are 3K eigenvalues P and 3K eigenvectors $\{\mathrm{X}\}$ that satisfy equation (30). The lowest positive eigenvalue P is the critical buckling pressure, P_{cr}, and the corresponding eigenvector $\{\mathrm{X}\}$ defines the buckling mode shape, i.e., the normalized values of the U_i, V_j, and W_k undetermined coefficients in equations (2), the assumed buckled shape.

The terms in $[a]$ and $[b]$ were obtained by substituting equations (2) into (25) and solving the various combinations of multiple partial differentiations, integrations, and summations and the results are provided here:

$$a_{[3(i-1)+1],[3(i-1)+1]} = \frac{C_x \pi^3 r m_i^{\,2}}{4L} + \frac{G_{LT}\pi t L n^2 k_2}{4r} + \frac{\pi n^2 N}{2r_c^{\,3}}(n^2 E_f I_x + G_f J) \text{, for } i = 1, K$$

where $m_i = n_i(N+1)$, $n_i = 2(i-1)+1$

$$b_{[3(i-1)+1],[3(i-1)+1]} = \frac{\pi L n^2}{4}\left(1 - \frac{\gamma_s^{\,e}}{r}\right) + \frac{E_f \pi A_f \gamma_f^{\,e} N n^2}{2t}\left(1 + \frac{e}{r}\right) \text{, for } i = 1, K$$

$$a_{[3(r-1)+1],[3(s-1)+1]} = \frac{\pi n^2 N}{4r_c^{\,3}}(n^2 E_f I_x + G_f J) \text{, for } r, s = 1, K \text{ but } r \neq s$$

$$b_{[3(r-1)+1],[3(s-1)+1]} = 0 \text{, for } r, s = 1, K \text{ but } r \neq s$$

$$a_{[3(i-1)+2],[3(i-1)+2]} = \frac{C_\phi \pi L n^2}{4r} + \frac{G_{LT}\pi^3 m_i^{\,2} t r k_2}{4L} \text{, for } i = 1, K$$

$$b_{[3(i-1)+2],[3(i-1)+2]} = \frac{\pi^3 (r m_i \alpha)^2}{8L} \text{, for } i = 1, K$$

$$a_{[3(i-1)+3],[3(i-1)+3]} = \frac{\pi L}{4r}\left\{C_\phi + \frac{D_\phi}{r^2}(n^2-1)^2 + \frac{D_x \pi^2 m_i^2}{L^2}\left[\frac{(\pi m_i r)^2}{L^2} + 2\upsilon_{\phi x}(n^2-1)\right]\right\}$$
$$+\frac{m_i^2 \pi^3 N}{2r_c^3 L^2}\left\{\left[e_1 n^2(e_1 n^2 - 2r_c) + r_c^2\right]E_f I_x + r^2 n^2 G_f J\right\}$$
$$+\frac{G_{LT}(\pi t)^3 (m_i n)^2}{12\mathrm{rL}}, \text{ for } i = 1, K$$

$$b_{[3(i-1)+3],[3(i-1)+3]} = \frac{\pi L}{4}\left[\frac{(\pi r m_i \alpha)^2}{2L^2} + \frac{n^2}{r}(r - \gamma_s^{\ e}) - 1\right], \text{ for } i = 1, K$$

$$a_{[3(r-1)+3],[3(s-1)+3]} = \frac{n_r n_s \pi^3 (N+1)^2 N}{4r_c^3 L^2}\left\{\left[e_1 n^2(e_1 n^2 - 2r_c) + r_c^2\right]E_f I_x + r^2 n^2 G_f J\right\}$$

$, \text{ for } r, s = 1, K$ but $r \neq s$ and where

$n_r = 2(r-1)+1$, $n_s = 2(s-1)+1$

$b_{[3(r-1)+3],[3(s-1)+3]} = 0$, for $r, s = 1, K$ but $r \neq s$

$$a_{[3(i-1)+1],[3(i-1)+2]} = -\frac{\pi^2 m_i n}{4}(G_{LT} t k_3 + \upsilon_{\phi x} C_x) = a_{[3(i-1)+2],[3(i-1)+1]}, \text{ for } i = 1, K$$

$$b_{[3(i-1)+1],[3(i-1)+2]} = \frac{\pi^2 r m_i n}{8} = b_{[3(i-1)+2],[3(i-1)+1]}, \text{ for } i = 1, K$$

$a_{[3(r-1)+1],[3(s-1)+2]} = a_{[3(r-1)+2],[3(s-1)+1]} = 0$, for $r, s = 1, K$ but $r \neq s$

$b_{[3(r-1)+1],[3(s-1)+2]} = b_{[3(r-1)+2],[3(s-1)+1]} = 0$, for $r, s = 1, K$ but $r \neq s$

$$a_{[3(i-1)+1],[3(i-1)+3]} = \frac{\pi^2 m_i}{4}\left(\upsilon_{\phi x} C_x - \frac{G_{LT} t^3 n^2}{6r^2}\right) + \frac{\pi^2 m_i n^2 N}{2L r_c^3}\left[(e_1 n^2 - r_c)E_f I_x - r G_f J\right]$$
$$= a_{[3(i-1)+3],[3(i-1)+1]}, \text{ for } i = 1, K$$

$$b_{[3(i-1)+1],[3(i-1)+3]} = -\frac{\pi^2 r m_i}{4} = b_{[3(i-1)+3],[3(i-1)+1]}, \text{ for } i = 1, K$$

$$a_{[3(i-1)+1],[3(k-1)+3]} = \frac{\pi^2 n_k n^2 (N+1) N}{2L r_c^3}\left[(e_1 n^2 - r_c)E_f I_x - r G_f J\right]$$
$$= a_{[3(k-1)+3],[3(i-1)+1]}, \text{ for } i, k = 1, K \text{ but } i \neq k \text{ and where}$$

$n_k = 2(k-1)+1$

$b_{[3(i-1)+1],[3(k-1)+3]} = 0 = b_{[3(k-1)+3],[3(i-1)+1]}$, for $i, k = 1, K$ but $i \neq k$

$$a_{[3(i-1)+2],[3(i-1)+3]} = -\frac{\pi n}{4r}\left(C_\phi L + \frac{G_{LT}\pi^2 t^3 m_i^{\ 2}}{6L}\right) = a_{[3(i-1)+3],[3(i-1)+2]}\text{ , for } i = 1, K$$

$$b_{[3(i-1)+2],[3(i-1)+3]} = \frac{\pi n L t \gamma_s^{\ e}}{4rt} = b_{[3(i-1)+3],[3(i-1)+2]}\text{ , for } i = 1, K \tag{31}$$

$$a_{[3(r-1)+2],[3(s-1)+3]} = a_{[3(r-1)+3],[3(s-1)+2]} = 0\text{ , for } r, s = 1, K \text{ but } r \neq s$$

$$b_{[3(r-1)+2],[3(s-1)+3]} = b_{[3(r-1)+3],[3(s-1)+2]} = 0\text{ , for } r, s = 1, K \text{ but } r \neq s.$$

In the terms for $[a]$ and $[b]$ above, the following definitions are needed:

$$k_2 = 1 + \frac{t^2}{12r^2}$$

$$k_3 = 1 - \frac{t^2}{12r^2} \tag{32}$$

Also, Figures 2 and 3 defined several of the geometric variables. A_f is the area of one ring, I_x is the moment of inertia of one ring about a radial line through the ring centroid, J is the ring torsion constant, and E_f is the ring frame elastic modulus. The variables $\gamma_s^{\ e}$, α , and $\gamma_f^{\ e}$ were defined in equations (6), (7), and (10), respectively. It is important to note that when applying the pseudo-plastic methods of DAPS4 (discussed later), only elastic material properties must be used in the definitions of $\gamma_s^{\ e}$ and $\gamma_f^{\ e}$. The ring eccentricities, e and e_1, were described earlier.

Finally, the equations are theoretically exact for an isotropic shell or for a specially orthotropic shell with a 0-90 balanced arrangement in which the laminated shell constitutive equations can be represented by

$$\begin{Bmatrix} N_\phi \\ N_x \\ N_{\phi x} \\ M_\phi \\ M_x \\ M_{\phi x} \end{Bmatrix} = \begin{bmatrix} C_\phi & \upsilon_{\phi x} C_x & 0 & 0 & 0 & 0 \\ \upsilon_{x\phi} C_\phi & C_x & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_\phi & \upsilon_{\phi x} D_x & 0 \\ 0 & 0 & 0 & \upsilon_{x\phi} D_\phi & D_x & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{\ 0} \\ \varepsilon_y^{\ 0} \\ \gamma_{xy}^{\ 0} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{33}$$

where all bending and stretching is completely uncoupled and provided G_{LT} can be defined as

$$G_{LT} = C_{66} / t = 12 D_{66} / t^3. \tag{34}$$

If theoretical homogeneity is achieved in the laminate, then C_ϕ, C_x, D_ϕ, and D_x can be represented by the relations

$$C_\phi = \frac{E_\phi t}{1-\upsilon_{\phi x}\upsilon_{x\phi}}$$
$$C_x = \frac{E_x t}{1-\upsilon_{\phi x}\upsilon_{x\phi}}$$
$$D_\phi = \frac{E_\phi t^3}{12(1-\upsilon_{\phi x}\upsilon_{x\phi})} \qquad (35)$$
$$D_x = \frac{E_x t^3}{12(1-\upsilon_{\phi x}\upsilon_{x\phi})}$$

where E_ϕ and E_x are the hoop (shell circumferential - major) and axial (shell longitudinal - minor) elastic moduli in the shell, respectively, and $\upsilon_{\phi x}$ and $\upsilon_{x\phi}$ are the major and minor Poisson ratios, respectively.

For an isotropic material, the shell constitutive constants reduce to

$$C_\phi = C_x = \frac{Et}{1-\upsilon^2}$$
$$D_\phi = D_x = \frac{Et^3}{12(1-\upsilon^2)}$$
$$C_{66} = Gt \qquad (36)$$
$$D_{66} = \frac{Et^3}{24(1+\upsilon)}$$
$$G_{LT} = G = \frac{E}{2(1+\upsilon)}$$

where E = Young's modulus, υ = Poisson's ratio, G = the shear modulus, and t = the shell thickness.

In example problems calculated by the DAPS4 program, satisfactory convergence is usually obtained with $K = 4$; the program allows a maximum value of $K = 8$. The number of circumferential waves, n, in the buckled shape corresponding to the critical buckling pressure, P_{cr}, will be lower for a long bay length, with n = 2 in the limit, such as for a long monocoque shell. For interbay buckling of a shell with closely spaced rings, n could be as high as 7 or 8 or even higher. The n = 0 mode is an axisymmetric failure mode and is a combined yielding and buckling phenomenon that will be treated separately in a later section using another methodology entirely.

General Instability

General instability is a non-symmetric collapse of the full length of the shell, i.e., buckling of the rings and shell plating together. Figure 4 shows an example of this type of buckling. The solution for this problem was derived in reference 1 using essentially the same procedure as described in the preceding section for interbay buckling except that a simpler assumed buckled shape could be used:

$$u = A_1 \cos n\phi \cos \frac{m\pi x}{L}$$
$$v = B_1 \sin n\phi \sin \frac{m\pi x}{L} + B_2 \sin n\phi \left(1 - \cos \frac{2\pi x}{L_f}\right) \tag{37}$$
$$w = C_1 \cos n\phi \sin \frac{m\pi x}{L} + C_2 \cos n\phi \left(1 - \cos \frac{2\pi x}{L_f}\right)$$

where, for the solution of the general instability problem, m = 1 and the geometry variables x, L, and L_f are in accordance with Figure 3. With only one half sine wave (m = 1) suitable to describe the buckled shape over the entire length of the shell, the out-of-plane bending and torsion of the rings are insignificant; thus the simpler assumed buckled shape is possible. The B_2 and C_2 terms in the expression for v and w in equations (37) serve to reduce the shell deflection at the frames due to their compressive and in plane bending stiffness.

Figure 4. Non-Symmetric General Buckling

In accordance with equations (4) through (11), the total strain and work energy accounted for in DAPS3 and DAPS4 for the general instability failure mode is:

$$U_T = U_e + U_b + V_e + V_b + W_p \tag{38}$$

or

$$
\begin{aligned}
U_T &= \frac{r}{2}\int_0^{2\pi}\int_0^L [C_x u_x{}^2 + \frac{C_\phi}{r^2}(v_\phi - w)^2 + 2\upsilon_{\phi x} C_x \frac{u_x}{r}(v_\phi - w)] d\phi dx \\
&+ \frac{G_{LT} t r}{2}\int_0^{2\pi}\int_0^L (\frac{u_\phi}{r} + v_x)^2 d\phi dx \\
&- \frac{\Pr^2 \alpha^2}{4}\int_0^{2\pi}\int_0^L (v_x{}^2 + w_x{}^2) d\phi dx \\
&+ \frac{P(\overset{e}{\gamma}_s - r)}{2r}\int_0^{2\pi}\int_0^L (u_\phi{}^2 + w_\phi{}^2 - 2wv_\phi) d\phi dx \\
&+ \frac{1}{2r}\int_0^{2\pi}\int_0^L [D_x r^2 w_{xx}{}^2 + \frac{D_\phi}{r^2}(w_{\phi\phi} + w)^2 + 2\upsilon_{\phi x} D_x w_{xx}(w_{\phi\phi} + w)] d\phi dx \\
&+ \frac{G_{LT} t^3}{6r}\int_0^{2\pi}\int_0^L (w_{x\phi} + \frac{v_x}{2} - \frac{u_\phi}{2r})^2 d\phi dx \\
&+ \frac{E_f A_f}{2r}\int_0^{2\pi} [(w_{\phi\phi} + w)\frac{e}{r} - (v_\phi - w)]^2 d\phi \,,\, x = aL_f \,,\, \sum_{a=1}^{N} \\
&+ \frac{P E_f A_f \overset{e}{\gamma}_f}{t}\int_0^{2\pi} [(w_{\phi\phi} + w)\frac{e}{r} - (v_\phi - w)] v_\phi d\phi \,,\, x = aL_f \,,\, \sum_{a=1}^{N} \\
&- \frac{P E_f A_f \overset{e}{\gamma}_f (1 + \frac{e}{r})}{2t}\int_0^{2\pi} (u_\phi{}^2 + w_\phi{}^2 - 2wv_\phi) d\phi \,,\, x = aL_f \,,\, \sum_{a=1}^{N} \\
&+ \frac{E_f I_r}{2r^3}\int_0^{2\pi} (w_{\phi\phi} + w)^2 d\phi \,,\, x = aL_f \,,\, \sum_{a=1}^{N} \\
&- \frac{\Pr}{2}\int_0^{2\pi}\int_0^L (2wu_x + \frac{2wv_\phi}{r} - v_\phi u_x - \frac{w^2}{r}) d\phi dx.
\end{aligned}
\tag{39}
$$

Upon substitution of equations (37) into (39) and then minimizing the total energy with respect to the five undetermined coefficients by taking the following partials

$$\frac{\partial U_T}{\partial A_1} = \frac{\partial U_T}{\partial B_1} = \frac{\partial U_T}{\partial C_1} = \frac{\partial U_T}{\partial B_2} = \frac{\partial U_T}{\partial C_2} = 0 \,, \tag{40}$$

and then segregating the terms that include the pressure, *P*, from those not involving *P*, then the system of equations can be written as

$$[a]\{\mathrm{X}\} - P[b]\{\mathrm{X}\} = \{0\} \tag{41}$$

where

$[a]$ and $[b]$ are square matrices each of order 5 x 5 and

$\{\mathrm{X}\} \equiv$ a vector of length 5 consisting of A_1, B_1, C_1, B_2, C_2.

There are 5 eigenvalues *P* and 5 eigenvectors $\{\mathrm{X}\}$ that satisfy equation (41). The lowest positive eigenvalue *P* is the critical buckling pressure, P_{cr}, and the corresponding eigenvector $\{\mathrm{X}\}$ defines the buckling mode shape, i.e.,

the normalized values of the A_1, B_1, C_1, B_2, C_2 undetermined coefficients in equations (37), the assumed buckled shape.

The terms in $[a]$ and $[b]$ were obtained by substituting equations (37) into (39) and solving the various combinations of multiple partial differentiations, integrations, and summations and the results are provided here:

$$a_{11} = \frac{C_x \pi^3 r m^2}{4L} + \frac{G_{LT} \pi t L n^2 k_2}{4r}$$

$$b_{11} = \frac{\pi L n^2}{4}\left(1 - \frac{\gamma_s{}^e}{r}\right) + \frac{E_f \pi A_f \gamma_f{}^e (N-1) n^2}{4t}\left(1 + \frac{e}{r}\right)$$

$$a_{22} = \frac{C_\phi \pi L n^2}{4r} + \frac{G_{LT} \pi^3 m^2 t r k_2}{4L} + \frac{E_f \pi A_f (N+1) n^2}{4r}$$

$$b_{22} = \frac{\pi^3 (r m \alpha)^2}{8L} + \frac{E_f \pi A_f \gamma_f{}^e (N+1) n^2}{2t}$$

$$a_{33} = \frac{\pi L}{4r}\left\{C_\phi + \frac{D_\phi}{r^2}(n^2-1)^2 + \frac{D_x \pi^2 m^2}{L^2}\left[\frac{(\pi m r)^2}{L^2} + 2\upsilon_{\phi x}(n^2-1)\right]\right\}$$
$$+ \frac{E_f \pi A_f (N+1)}{4r}\left\{1 + \frac{e}{r}\left[\frac{e}{r}(n^2-1)^2 + 2(1-n^2)\right]\right\}$$
$$+ \frac{G_{LT}(\pi t)^3 (mn)^2}{12\mathrm{rL}} + \frac{E_f \pi I_R (N+1)(n^2-1)^2}{4r^3}$$

$$b_{33} = \frac{\pi L}{4}\left[\frac{E_f A_f \gamma_f{}^e n^2}{tL}\left(1 + \frac{e}{r}\right)(N+1) + \frac{(\pi r m \alpha)^2}{2L^2} + \frac{n^2}{r}(r - \gamma_s{}^e) - 1\right]$$

$$a_{44} = \frac{3C_\phi \pi L n^2}{4r} + \frac{G_{LT} \pi^3 t r L k_2}{L_f{}^2}$$

$$b_{44} = \frac{\pi^3 L}{2}\left(\frac{r\alpha}{L_f}\right)^2$$

$$a_{55} = \frac{\pi L}{2r}\left\{\frac{3C_\phi}{2} + \frac{3D_\phi}{2r^2}(n^2-1)^2 + \frac{4D_x \pi^2}{L_f{}^2}\left[\frac{2(\pi r)^2}{L_f{}^2} + \upsilon_{\phi x}(n^2-1)\right]\right\}$$
$$+ \frac{G_{LT}(\pi t)^3 L n^2}{3\mathrm{rL_f}^2}$$

$$b_{55} = \frac{\pi^3 L (r\alpha)^2}{2L_f{}^2} - \frac{3\pi L}{4}\left[1 + n^2\left(\frac{\gamma_s{}^e}{r} - 1\right)\right]$$

$$a_{12} = -\frac{\pi^2 mn}{4}(G_{LT} t k_3 + \upsilon_{\phi x} C_x) = a_{21}$$

$$b_{12} = \frac{\pi^2 rmn}{8} = b_{21}$$

$$a_{13} = \frac{\pi^2 m}{4}\left(\upsilon_{\phi x} C_x - \frac{G_{LT} t^3 n^2}{6r^2}\right) = a_{31}$$

$$b_{13} = -\frac{\pi^2 rm}{4} = b_{31}$$

$$a_{14} = -\pi n k_1 (G_{LT} t k_3 + \upsilon_{\phi x} C_x) = a_{41}$$

$$b_{14} = \frac{\pi r n k_1}{2} = b_{41}$$

where $k_1 = \dfrac{2(N+1)^2(1-\cos m\pi)}{4(N+1)^2 - m^2}$

$$a_{15} = \pi k_1 \left(\upsilon_{\phi x} C_x - \frac{G_{LT} t^3 n^2}{6r^2}\right) = a_{51}$$

$$b_{15} = -\pi r k_1 = b_{51}$$

$$a_{23} = -\frac{\pi n}{4r}\left\{C_\phi L + \frac{G_{LT}\pi^2 t^3 m^2}{6L} + E_f A_f (N+1)\left[1 + \frac{e}{r}(1-n^2)\right]\right\} = a_{32}$$

$$b_{23} = \frac{\pi n}{4rt}\left\{Lt\gamma_s^{\ e} + E_f A_f r \gamma_f^{\ e}(N+1)\left[\frac{e}{r}(n^2-2) - 2\right]\right\} = b_{32}$$

$$a_{24} = \frac{C_\phi L n^2 k_1}{rm} + \frac{G_{LT}\pi^2 m t r k_1 k_2}{L} = a_{42}$$

$$b_{24} = \frac{k_1(\pi r \alpha)^2 m}{2L} = b_{42}$$

$$a_{25} = -\frac{C_\phi L n k_1}{rm} - \frac{G_{LT}\pi^2 t^3 m n k_1}{6rL} = a_{52}$$

$$b_{25} = \frac{L n k_1 \gamma_s^{\ e}}{rm} = b_{52}$$

$$a_{34} = -\frac{C_\phi L n k_1}{rm} - \frac{G_{LT}\pi^2 t^3 m n k_1}{6rL} = a_{43} = a_{25}$$

$$b_{34} = \frac{L n k_1 \gamma_s^{\ e}}{rm} = b_{43} = b_{25}$$

$$
\begin{aligned}
a_{35} &= \frac{\pi^2 k_1 m}{Lr}\left[D_x\left(\frac{m\pi r}{L}\right)^2 + 2\upsilon_{\phi x} D_x (n^2-1) + \frac{G_{LT} t^3 n^2}{3}\right] \\
&\quad + \frac{k_1 L}{rm}\left[C_\phi + \frac{D_\phi (n^2-1)^2}{r^2}\right] = a_{53} \\
b_{35} &= \frac{m k_1}{2L}(\pi r \alpha)^2 + \frac{k_1 L}{m}\left[n^2\left(1-\frac{\gamma_s^{\,e}}{r}\right)-1\right] = b_{53} \\
a_{45} &= -\frac{\pi L n}{2r}\left(\frac{3C_\phi}{2} + \frac{G_{LT}\pi^2 t^3}{3L_f^{\,2}}\right) = a_{54} \\
b_{45} &= \frac{3\pi L n \gamma_s^{\,e}}{4r} = b_{54}.
\end{aligned} \tag{42}
$$

In the definitions for the a_{ij} and b_{ij}, the variables and material properties have the same definitions as given for the solution to the interbay buckling problem given in the preceding section.

Stresses and Deflections

The solution for the stresses and deflections was derived for specially orthotropic, hybrid[2] ring-stiffened (or monocoque) cylindrical shells of revolution originally in reference 2 and subsequently re-presented in reference 1 in terms of the laminate constants of equation (33). The governing differential equation for the problem was derived from the equilibrium of forces and moments on a differential shell element and is provided here:

$$
D_x \frac{d^4 w}{dx^4} + \frac{PR\alpha^2}{2}\frac{d^2 w}{dx^2} + \frac{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}{R^2} w = -P\left(1-\frac{\upsilon_{\phi x}\alpha^2}{2}\right). \tag{43}
$$

The second term of equation (43) renders the solution to be nonlinear in the pressure.[3] In this equation, the axial x coordinate has its origin at mid-bay between two rings. The solution for the deflections and stresses at mid-bay and at a ring frame are provided as follows. Not all of these equations were provided in the prior publications or in DAPS3 but are included in DAPS4. (The various functions F_1, F_2, … η_1, η_2, etc., used in the definitions of the stresses and deflections are defined after all the definitions are given.)

The axisymmetric radial displacement w (positive outward) as a function of the axial coordinate x is:

[2] A hybrid shell refers to a shell in which the skin is made of one material and the ring stiffeners are made of another material. For example, the shell skin could be constructed of a laminated composite and the ring stiffeners constructed of aluminum. Conversely, the skin could be an isotropic material such as aluminum or steel and the rings made of a unidirectional fiber wound composite.

[3] This nonlinearity is sometimes referred to as a “beam-column” effect because it results from the interaction between the longitudinal bending and the longitudinal compression as a consequence of the axial portion of the hydrostatic pressure. The effect is usually small but can be accentuated in a very thin shell, particularly if it has orthotropic material properties with greater stiffness in the hoop direction than the axial direction.

$$w(x)=\frac{-PR^2}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}\left(1-\frac{\upsilon_{\phi x}\alpha^2}{2}\right)$$
$$+\frac{PR^2\beta}{\delta}\left(F_2\cosh\frac{2\theta\eta_1 x}{L_s}\cos\frac{2\theta\eta_2 x}{L_s}+F_5\sinh\frac{2\theta\eta_1 x}{L_s}\sin\frac{2\theta\eta_2 x}{L_s}\right) \tag{44}$$

where $\delta=E_f A_{eff}+C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})(b+L_sF_1)$ and

β was defined in equations (7).

Deflection at mid-bay, i.e., at $x=0$:

$$w_m=\frac{-PR^2}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}\left(1-\frac{\upsilon_{\phi x}\alpha^2}{2}\right)+\frac{PR^2\beta}{\delta}F_2. \tag{45}$$

Deflection at a frame, i.e., at $x=\frac{L_s}{2}$:

$$w_f=\frac{-PR^2}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}\left(1-\frac{\upsilon_{\phi x}\alpha^2}{2}\right)$$
$$+\frac{PR^2\beta}{\delta}\left(F_2\cosh\theta\eta_1\cos\theta\eta_2+F_5\sinh\theta\eta_1\sin\theta\eta_2\right). \tag{46}$$

The curvature as a function of *x* is the second derivative of equation (44):

$$w''(x)=-\frac{2PR^2\beta\theta^2}{L_s^{\,2}\delta}\left\{\begin{aligned}&\cosh\frac{2\theta\eta_1 x}{L_s}\cos\frac{2\theta\eta_2 x}{L_s}\left[\gamma F_2-4\eta_1\eta_2F_5\right]\\&+\sinh\frac{2\theta\eta_1 x}{L_s}\sin\frac{2\theta\eta_2 x}{L_s}\left[\gamma F_5+4\eta_1\eta_2F_2\right]\end{aligned}\right\}. \tag{47}$$

The expressions for stresses as a function of *x* were derived in reference 2 and given as equations (A-60) and (A-67) in that reference for the longitudinal and circumferential directions, respectively:[4]

[4]In the expressions for stress, σ_i^o, the superscript *o* and subscript *i* refer to the outer surface and inner surface, respectively, and correlate to the order of the addition and subtraction symbols, $\mp$ or $\pm$ in the equations, as the case may be.

$$\sigma_{x_i}^{o} = -\frac{PR\alpha^2}{2t} \mp \frac{6D_x}{t^2}\left(B\lambda_1^{\,2}\cosh\lambda_1 x + F\lambda_3^{\,2}\cosh\lambda_3 x\right)$$

$$\sigma_{\phi_i}^{o} = -\frac{PR}{t} + \left[\frac{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}{tR} \mp \frac{6\upsilon_{x\phi}D_\phi\lambda_1^{\,2}}{t^2}\right]B\cosh\lambda_1 x$$

$$+\left[\frac{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}{tR} \mp \frac{6\upsilon_{x\phi}D_\phi\lambda_3^{\,2}}{t^2}\right]F\cosh\lambda_3 x$$

where B and F were determined from boundary conditions to be :

$$B = -P\beta(\eta_1 - i\eta_2)(\sinh\eta_1\theta\cos\eta_2\theta - i\cosh\eta_1\theta\sin\eta_2\theta)/\Phi$$

$$F = P\beta(\eta_1 + i\eta_2)(\sinh\eta_1\theta\cos\eta_2\theta + i\cosh\eta_1\theta\sin\eta_2\theta)/\Phi$$

where β and θ were defined in equations (7) and

$$\Phi = 2i\eta_1\eta_2\left\{\begin{aligned}&\left[E_f A_{eff} + bC_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})\right]\Omega/R^2\\ &+16D_x(\theta/L_s)^3(\cosh^2\theta\eta_1 - \cos^2\theta\eta_2)\end{aligned}\right\}$$

$$\lambda_1 = \frac{2\theta}{L_s}(\eta_1 + i\eta_2),\ \lambda_3 = \frac{2\theta}{L_s}(\eta_1 - i\eta_2),\ i = \sqrt{-1}$$

and Ω is defined later, in equation (62). (48)

Letting $\zeta = \frac{2x}{L_s}$, i.e., $x = \frac{\zeta L_s}{2}$, then the stresses can be solved as a function of a fraction of $\frac{L_s}{2}$ so the range $0 \le \zeta \le 1$ (with $\zeta = 0$ at mid-bay) coincides with the range $0 \le x \le \frac{L_s}{2}$. Equations (48) are therefore solved to provide the following equations:

$$\sigma_{x_i}^{o}(\zeta) = \frac{\sigma_u\alpha^2}{2} \pm \frac{6\sigma_{\phi mf}}{t}\sqrt{\frac{D_x}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}}\begin{bmatrix}F_4\cosh\theta\eta_1\zeta\cos\theta\eta_2\zeta\\ -F_6\sinh\theta\eta_1\zeta\sin\theta\eta_2\zeta\end{bmatrix} \tag{49}$$

$$\sigma_{\phi_i}^{o}(\zeta) = \sigma_u - \sigma_{\phi mf}\left[F_2\cosh\theta\eta_1\zeta\cos\theta\eta_2\zeta + F_5\sinh\theta\eta_1\zeta\sin\theta\eta_2\zeta\right]$$
$$\pm\upsilon_{\phi x}\frac{6\sigma_{\phi mf}}{t}\sqrt{\frac{D_x}{C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi})}}\begin{bmatrix}F_4\cosh\theta\eta_1\zeta\cos\theta\eta_2\zeta\\ -F_6\sinh\theta\eta_1\zeta\sin\theta\eta_2\zeta\end{bmatrix} \tag{50}$$

where

$$\sigma_u = -\frac{PR}{t} \text{ and } \sigma_{\phi mf} = \frac{\sigma_u\beta}{\delta}C_\phi(1-\upsilon_{\phi x}\upsilon_{x\phi}) \tag{51}$$

and where δ was defined in equations (44).

The stresses of most interest are those at mid-bay ($\zeta = 0$) and at a frame ($\zeta = 1$), the solution of which are:

At mid-bay

$$\sigma_{x,m_i}^{o} = \frac{\sigma_u \alpha^2}{2} \pm \sigma_{xbm}$$
$$\sigma_{\phi,m_i}^{o} = \sigma_u - \sigma_{\phi mf} F_2 \pm \upsilon_{\phi x} \sigma_{xbm} \tag{52}$$

At a frame

$$\sigma_{x,f_i}^{o} = \frac{\sigma_u \alpha^2}{2} \pm \frac{6\sigma_{\phi mf} F_3}{t} \sqrt{\frac{D_x}{C_\phi (1-\upsilon_{\phi x}\upsilon_{x\phi})}}$$
$$\sigma_{\phi,f_i}^{o} = \sigma_u - \sigma_{\phi mf} \pm \upsilon_{\phi x} \frac{6\sigma_{\phi mf} F_3}{t} \sqrt{\frac{D_x}{C_\phi (1-\upsilon_{\phi x}\upsilon_{x\phi})}} \tag{53}$$

where

$$\sigma_{xbm} = \frac{6\sigma_u \beta F_4}{t\delta} \sqrt{D_x C_\phi (1-\upsilon_{\phi x}\upsilon_{x\phi})}\ . \tag{54}$$

The hoop stress in the frame is

$$\sigma_{\phi f} = \frac{Q * R}{A_{eff} + bt} \text{ where}$$
$$Q* = -Pb\left(1 - \frac{\upsilon_{x\phi}\alpha}{2}\right) + \sigma_{\phi mf} \frac{tL_s F_1}{R}. \tag{55}$$

The following variables are used in equations (44) through (55):

$$F_1 = \frac{4}{\theta}\left[\cosh^2 \theta\eta_1 - \cos^2 \theta\eta_2\right] \big/ \Omega \tag{56}$$

$$F_2 = \left[\frac{\cosh\theta\eta_1 \sin\theta\eta_2}{\eta_2} + \frac{\sinh\theta\eta_1 \cos\theta\eta_2}{\eta_1}\right] \Bigg/ \Omega \tag{57}$$

$$F_3 = \left[\frac{\cos\theta\eta_2 \sin\theta\eta_2}{\eta_2} - \frac{\cosh\theta\eta_1 \sinh\theta\eta_1}{\eta_1}\right] \Bigg/ \Omega \tag{58}$$

$$F_4 = \left[\frac{\cosh\theta\eta_1 \sin\theta\eta_2}{\eta_2} - \frac{\sinh\theta\eta_1 \cos\theta\eta_2}{\eta_1}\right] \Bigg/ \Omega \tag{59}$$

$$F_5 = \left[\frac{\cosh\theta\eta_1 \sin\theta\eta_2}{\eta_1} - \frac{\sinh\theta\eta_1 \cos\theta\eta_2}{\eta_2}\right] \Bigg/ \Omega \tag{60}$$

$$F_6 = \left[\frac{\cosh\theta\eta_1 \sin\theta\eta_2}{\eta_1} + \frac{\sinh\theta\eta_1 \cos\theta\eta_2}{\eta_2}\right] \Bigg/ \Omega \tag{61}$$

$$\Omega = \frac{\cosh\theta\eta_1 \sinh\theta\eta_1}{\eta_1} + \frac{\cos\theta\eta_2 \sin\theta\eta_2}{\eta_2} \tag{62}$$

$$\eta_1 = \frac{1}{2}\sqrt{1-\gamma}\ ,\ \eta_2 = \frac{1}{2}\sqrt{1+\gamma} \tag{63}$$

$$\gamma = \frac{PR^2\alpha^2}{4\sqrt{D_x C_\phi (1-\upsilon_{\phi x}\upsilon_{x\phi})}} \tag{64}$$

and θ was defined in equations (7). The above solution is valid for $\gamma < 1$.

Axisymmetric Collapse

Axisymmetric collapse is a combined yielding and buckling phenomenon that is precipitated by axisymmetric yielding in the shell plating between two rings (a bay); collapse occurs when three plastic hinges form: at mid-bay and at the two rings bordering that bay. Figure 5 shows an example of this type of buckling. Over the years, test data for the axisymmetric mode of collapse for shells made of isotropic materials have correlated best with this stress-based approach to calculation rather than other options such as computing the interbay buckling with equation (30) and setting $n = 0$.

Figure 5. Axisymmetric Local Buckling

For isotropic shells, the theory for the stresses and deflections documented in the preceding section reduce to the theory of Pulos and Salerno of reference 4. DAPS3 computes yielding based on equations (52) and the Hencky-Von Mises yield criterion at the outer surface at mid-bay to indicate the onset of axisymmetric collapse.[5] This methodology was recommended in reference 5, based on good correlations with experiment, though in some cases, the result indicated the methodology to be conservative by more than 10%.

[5]For orthotropic shells, the Von Mises yield criterion does not apply so, by default, DAPS3 and DAPS4 use a maximum stress yield criterion at the outer fiber mid-bay point as a conservative safe limit calculation of this mode of collapse.

The plastic reserve strength between the initiation of yielding and failure (e.g., collapse or "fast fracture") can be appreciable and should be taken into account. Lunchick took this into account in reference 6 for the axisymmetric collapse problem of a stiffened cylinder under pressure and obtained good agreement with test results. That particular problem was based on the formation of a third fully plastic hinge at mid-bay, through the thickness, and with a ductile material.

DAPS4 takes advantage of an improvement to the DAPS3 methodology by incorporating Lunchick's theory (reference 6) for determining the plastic reserve strength in the shell in the formation of three fully plastic hinges in the computation of the axisymmetric collapse mode (applicable to isotropic materials only). The improvement tended to remove much of the conservatism of those conservative cases mentioned above; at the other extreme, the maximum degree of un-conservatism in the study was 3% when incorporating the plastic reserve strength.

The stress resultants in the derivation of reference 6 were assumed to be linear with the loading even into the plastic range. This is considered reasonable when the loading into the plastic range at failure is not too much greater than the loading associated with the initiation of yielding. The analysis assumed an elastic-perfectly plastic material stress-strain relationship. Although the work-hardening capacity of a material may permit the applied pressure to be increased still further, this factor was not taken into account for conservatism and ease in calculation.

Figure 6 is redrawn from reference 6 to illustrate the progressive changes in stress distributions through the thickness of a cylindrical shell. Essentially, the derivation led to computing a dimensionless factor, ϕ_3, which represents a multiplier of the pressure as computed by DAPS3 for the axisymmetric collapse pressure; the multiplier ϕ_3 will be greater than or equal to 1.0, depending on the amount of the plastic reserve strength in forming the third plastic hinge.

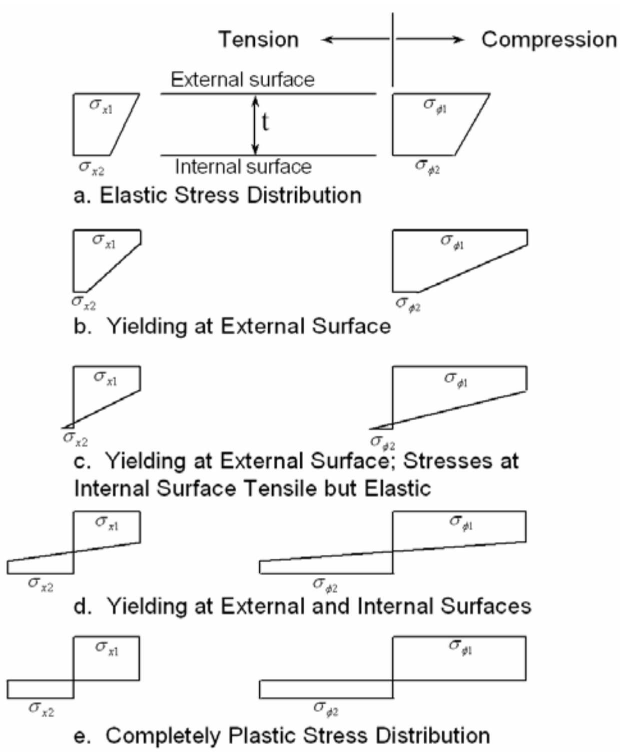


Figure 6. Progressive Changes in Stress Distributions Through Thickness of Cylindrical Shell

The axisymmetric collapse pressure (including the plastic reserve strength), P_c, is computed in DAPS4 from the following equation:

$$P_c = \phi_3 P_y \tag{65}$$

where P_y = the axisymmetric collapse pressure as computed by DAPS3, i.e., the pressure at which the outer surface at mid-bay begins to yield (in accordance with the Von-Mises yield criterion), and

$$\phi_3 = \sqrt{\frac{1+36\phi_1+12\phi_2}{1+8\phi_1+4\sqrt{4\phi_1^{\,2}+\phi_2^{\,2}}}} \tag{66}$$

$$\phi_1 = \frac{\vartheta_1}{\vartheta_4}, \;\; \phi_2 = \frac{\vartheta_2}{\vartheta_4}$$

$$\vartheta_1 = B_\phi^{\,2} - B_x B_\phi\left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right) + B_x^{\,2}\left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right)^2$$

$$\vartheta_2 = B_\phi - \frac{1}{2}(B_\phi + B_x)\left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right) + B_x\left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right)^2$$

$$\vartheta_4 = 1 - \left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right) + \left(\frac{\sigma_{mx}}{\sigma_{m\phi}}\right)^2$$

$$B_\phi = \frac{1}{6}\left(\frac{\sigma_{b\phi}}{\sigma_{m\phi}}\right)$$

$$B_x = \frac{B_\phi}{\upsilon\left(\dfrac{\sigma_{mx}}{\sigma_{m\phi}}\right)}$$

$$\sigma_{mx} = \frac{\sigma_u \alpha^2}{2}$$

$$\sigma_{m\phi} = \sigma_u - \sigma_{\phi mf} F_2$$

$$\sigma_{b\phi} = \upsilon \sigma_{xbm}$$

and where σ_u, $\sigma_{\phi mf}$, σ_{xbm}, and F_2 are computed with the help of equations (51), (54), and (57) using isotropic material properties. The solution for P_c in equation (65) is iterative because the stresses are nonlinear in the pressure P.

Pseudo-Plastic Method

The method DAPS4 uses to represent inelastic behavior in the buckling solutions is a very simple, yet effective, intuitive method used for almost 50 years in this type of analysis. The method dates back at least to its evaluation by Reynolds at the David Taylor Model Basin in reference 7. Put simply, the inelastic buckling pressure (whether using the interbay buckling equations or the general instability equations in the above sections) is computed recognizing that the collapse pressure depends wholly on the stiffness of the shell plating and the rings.

Individually, the terms in the buckling solutions use the instantaneous (or "tangent") moduli corresponding to the stress level in the shell or rings at the computed pressure.

This apparent circular reference between the *pressure - stress level - tangent moduli - buckling pressure* can be more readily and visually represented by the graphical intersection of two curves (and illustrated in Figure 7): 1. Applied pressure as a function of resulting stresses (the red curve), and 2. Computed buckling pressure as a function of stresses (the blue curve). In the first curve, the stresses are actually calculated as a function of the applied pressure using the equations in the above section, "Stresses and Deflections", and then plotted in the reverse relationship. In the second curve, the buckling pressure is computed indirectly as a function of the stresses by using the tangent moduli in the shell and ring terms; the tangent moduli are computed from the slope of the stress-strain curves at the stresses that were computed in the shell and rings, respectively, at the pressure from the first curve. The above substitutions are only valid (and logical) at the intersection of the two curves; it is the pressure at this intersection point that is taken as the inelastic buckling pressure.

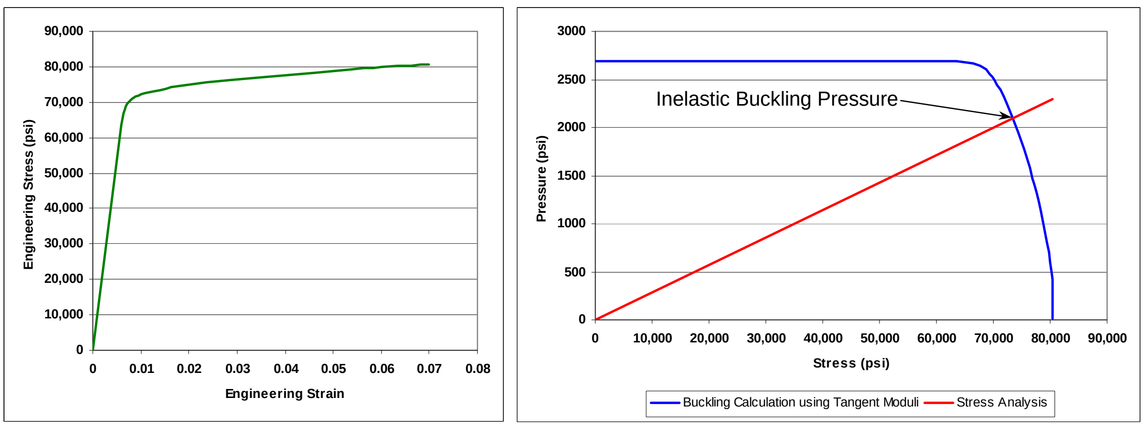


Figure 7. Illustration of the Pseudo-Plastic Method

In the case of isotropic shells, the stress level that is representative of the shell plating is the Von Mises stress at the mid-bay, mid-plane point. The tangent modulus for the shell terms is computed from the stress-strain curve at this stress level. The stress level in the ring is the ring stress computed at the same pressure used for computing the mid-bay, mid-plane Von Mises shell stress. The tangent modulus for the ring terms is computed from the stress-strain curve at this ring stress level. Terms involving the shear modulus are not reduced.

In the case of orthotropic shells, two tangent moduli are used in the shell terms: the longitudinal tangent modulus and the circumferential tangent modulus. These two moduli correspond to the computed longitudinal and circumferential stresses, respectively, at the mid-bay, mid-plane point. The tangent modulus for the ring terms is calculated from the ring stress computed at the same pressure used for computing the shell stresses.

Often the analyst obtains a typical stress strain curve for the material but wishes to design the pressure vessel to the minimum yield strength. Since the tangent modulus rapidly drops off in the knee of the curve, the computed inelastic buckling pressures are very sensitive to the yield strength of the material. DAPS4 accommodates this variation in material yield by providing an option to effectively shift the input stress strain curve by the ratio of the design yield stress to the nominal yield stress measured off the input curve. Figure 8 illustrates shifting the stress strain curve of Figure 7 to one with a reduced yield stress. All stress and strain points are multiplied by the ratio of the reduced yield stress to the nominal yield stress measured on the original curve. The elastic modulus remains unchanged but the proportional limit is reduced.

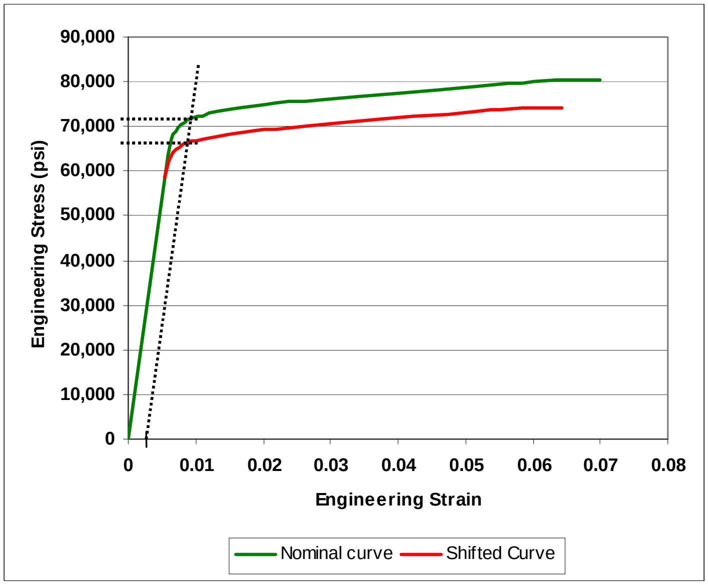


Figure 8. Illustration of Shifting the Stress Strain Curve

Correlation With Tests and the BOSOR5 Code

Boichot and Reynolds, in reference 8, documented 69 tests of small-scale ring-stiffened cylinders machined from 7075-T6 aluminum bar stock with a nominal yield strength of 80,000 psi. A schematic of these shells is given in Figure 9. Each specimen had six external rectangular rings, with the middle four rings uniformly spaced; the first and sixth rings were positioned to provide a reduced frame spacing in the two bays at each end as indicated in the figure. The dimensions (inches) and measured yield stress (compressive 0.2% offset, psi) of all the test models are given in Table I. Young's modulus and Poisson's ratio were reported to be 10.8E+06 psi and 0.3, respectively.

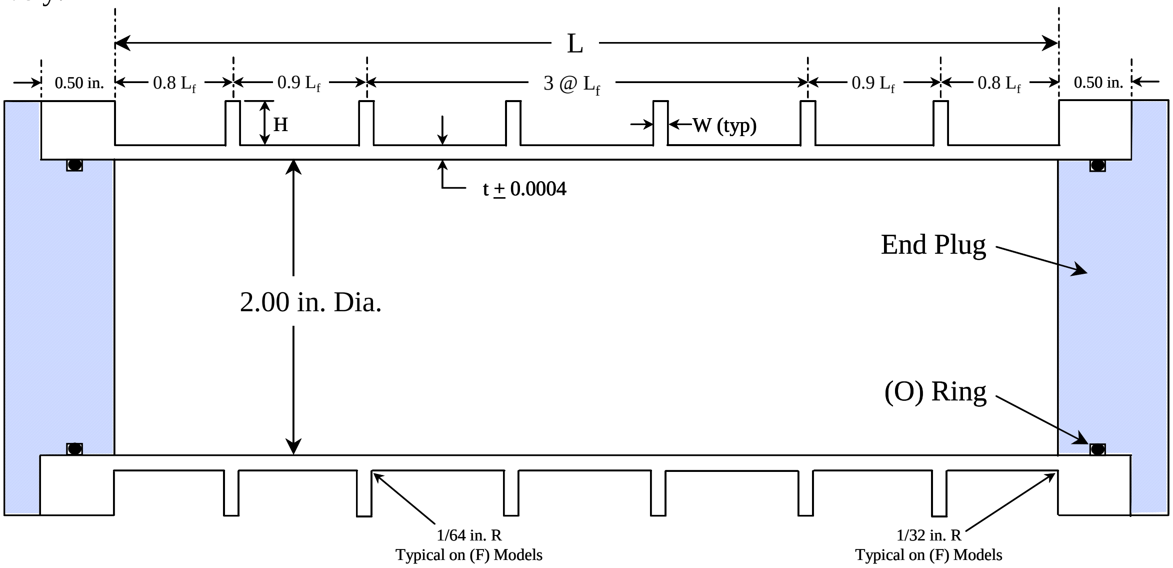


Figure 9. Geometry of All 69 Test Specimens

Table I also gives several dimensionless geometric parameters for each model: θ, r/t, $A_f/(tL_f)$, L/r, and L_f/r, where θ is given in equations (7) which, for isotropic materials, reduces to $\theta = \frac{\sqrt[4]{3(1-\upsilon^2)}L_s}{\sqrt{rt}}$. The naming convention for the test models adopted by Boichot and Reynolds was the first two digits represented 10θ, the third digit represented $100t/r$, and the fourth digit represented $10A_f/(tL_f)$. The first 45 models were machined without any fillets transitioning the ring edges to the shell hull surface. The last 24 models, designated with an F in the name, were machined with fillets included at the edges of all rings, as indicated in Figure 9. All 69 models were subjected to external hydrostatic pressure to collapse.

All 69 test models were analyzed with the DAPS4 code. Since DAPS4 is based on analysis of shells with uniformly spaced stiffeners down the entire length of the shell and the test models had reduced frame spacing near the ends, two analysis sets had to be done. For the first set of calculations, the *average* ring spacing, L_f, was input along with the actual shell length to best represent the overall shell stiffness (including the correct number of rings) for calculation of the general instability pressure. For the second set of calculations, the *actual* ring spacing, L_f, was input along with a longer shell length (L_f x the number of bays) to best represent the local ring-shell stiffness for calculation of the interbay buckling pressure, axisymmetric collapse pressure, and the stresses and deflections from mid-bay to the ring. The stress-strain curves were derived from the stress-strain data provided in reference 9. The complete input deck for both sets of calculations is given in Appendix A. Both data sets were stacked into a single run (138 cases) and execution took 23 seconds on a personal computer with a 3 GHz processor.

Table I. Test Specimen Dimensions and Yield Strengths

	Model	L_f	W	t	H	L	yield	OD	Theta	r/t	$A_f/(tL_f)$	L/r	L_f/r
1	25-88	0.699	0.127	0.0830	0.386	4.473	80600	2.1660	2.50	12.55	0.84	1.074	0.168
2	25-86	0.679	0.107	0.0832	0.318	4.349	80600	2.1664	2.50	12.52	0.60	1.044	0.163
3	25-84	0.659	0.086	0.0830	0.255	4.216	80700	2.1660	2.51	12.55	0.40	1.012	0.158
4	25-82	0.632	0.059	0.0826	0.179	4.047	80700	2.1652	2.51	12.61	0.20	0.972	0.152
5	20-88	0.572	0.112	0.0830	0.340	3.647	80700	2.1660	2.01	12.55	0.80	0.875	0.137
6	20-86	0.555	0.097	0.0830	0.287	3.552	80700	2.1660	2.00	12.55	0.60	0.853	0.133
7	20-84	0.537	0.079	0.0830	0.230	3.430	81400	2.1660	2.00	12.55	0.41	0.823	0.129
8	20-82	0.511	0.054	0.0832	0.158	3.280	82000	2.1664	2.00	12.52	0.20	0.787	0.123
9	15-88	0.444	0.100	0.0830	0.297	2.843	82700	2.1660	1.50	12.55	0.81	0.682	0.107
10	15-86	0.428	0.084	0.0830	0.253	2.745	82700	2.1660	1.50	12.55	0.60	0.659	0.103
11	15-84	0.413	0.068	0.0830	0.202	2.638	83000	2.1660	1.51	12.55	0.40	0.633	0.099
12	15-82	0.390	0.046	0.0830	0.140	2.502	83300	2.1660	1.50	12.55	0.20	0.601	0.094
13	10-88	0.313	0.084	0.0833	0.248	2.002	83500	2.1666	1.00	12.50	0.80	0.480	0.075
14	10-86	0.300	0.071	0.0830	0.211	1.920	83500	2.1660	1.00	12.55	0.60	0.461	0.072
15	10-84	0.286	0.057	0.0830	0.167	1.830	83400	2.1660	1.00	12.55	0.40	0.439	0.069
16	10-82	0.269	0.039	0.0833	0.115	1.715	83200	2.1666	1.00	12.50	0.20	0.412	0.065
17	25-58	0.532	0.086	0.0513	0.254	3.405	83000	2.1026	2.50	19.99	0.80	0.830	0.130
18	25-56	0.520	0.074	0.0510	0.217	3.327	83000	2.1020	2.51	20.11	0.61	0.811	0.127
19	25-54	0.506	0.060	0.0513	0.174	3.239	83200	2.1026	2.50	19.99	0.40	0.789	0.123
20	25-52	0.487	0.042	0.0511	0.123	3.117	83400	2.1022	2.50	20.07	0.21	0.760	0.119
21	20-58	0.434	0.077	0.0513	0.232	2.778	83600	2.1026	2.00	19.99	0.80	0.677	0.106
22	20-56	0.424	0.067	0.0513	0.194	2.715	83600	2.1026	2.00	19.99	0.60	0.662	0.103
23	20-54	0.410	0.053	0.0511	0.159	2.624	83400	2.1022	2.00	20.07	0.40	0.640	0.100
24	20-52	0.394	0.037	0.0511	0.110	2.522	83300	2.1022	2.00	20.07	0.20	0.615	0.096
25	15-58	0.337	0.069	0.0506	0.201	2.157	83200	2.1012	1.51	20.26	0.81	0.526	0.082
26	15-56	0.326	0.058	0.0513	0.174	2.085	83200	2.1026	1.50	19.99	0.60	0.508	0.079
27	15-54	0.315	0.047	0.0513	0.138	2.017	83400	2.1026	1.50	19.99	0.40	0.492	0.077
28	15-52	0.300	0.032	0.0514	0.097	1.919	83500	2.1028	1.50	19.96	0.20	0.468	0.073
29	10-58	0.235	0.057	0.0515	0.169	1.506	83700	2.1030	1.00	19.92	0.80	0.367	0.057
30	10-56	0.226	0.048	0.0511	0.146	1.446	83700	2.1022	1.00	20.07	0.61	0.352	0.055
31	10-54	0.217	0.040	0.0516	0.114	1.389	83800	2.1032	0.99	19.88	0.41	0.339	0.053
32	10-52	0.205	0.027	0.0518	0.078	1.311	84100	2.1036	0.99	19.81	0.20	0.319	0.050
33	25-28	0.320	0.043	0.0200	0.123	2.049	78400	2.0400	2.51	50.50	0.83	0.507	0.079
34	25-26	0.314	0.036	0.0204	0.106	2.008	80600	2.0408	2.49	49.52	0.60	0.497	0.078
35	25-24	0.307	0.030	0.0202	0.086	1.965	84300	2.0404	2.49	50.00	0.42	0.486	0.076
36	20-28	0.260	0.038	0.0203	0.111	1.661	84300	2.0406	1.99	49.76	0.80	0.411	0.064
37	20-26	0.253	0.032	0.0204	0.096	1.626	84400	2.0408	1.98	49.52	0.60	0.402	0.063
38	20-24	0.248	0.026	0.0210	0.076	1.586	84400	2.0420	1.96	48.12	0.38	0.392	0.061
39	15-28	0.200	0.033	0.0201	0.099	1.280	84000	2.0402	1.51	50.25	0.81	0.317	0.050
40	15-26	0.196	0.029	0.0205	0.081	1.254	83800	2.0410	1.49	49.28	0.58	0.310	0.049
41	15-24	0.190	0.023	0.0206	0.066	1.216	83500	2.0412	1.49	49.04	0.39	0.301	0.047
42	10-28	0.140	0.028	0.0206	0.079	0.888	83500	2.0412	1.00	49.04	0.77	0.220	0.035
43	10-26	0.135	0.024	0.0205	0.068	0.863	83500	2.0410	0.99	49.28	0.59	0.214	0.033
44	10-24	0.130	0.019	0.0203	0.056	0.834	83600	2.0406	1.00	49.76	0.40	0.206	0.032
45	10-22	0.124	0.014	0.0203	0.039	0.793	83600	2.0406	0.99	49.76	0.22	0.196	0.031
46	15-58F	0.337	0.067	0.0511	0.200	2.157	82000	2.1022	1.52	20.07	0.78	0.526	0.082
47	15-56F	0.325	0.056	0.0507	0.174	2.087	82000	2.1014	1.52	20.22	0.59	0.509	0.079
48	15-54F	0.313	0.042	0.0512	0.134	2.018	82000	2.1024	1.52	20.03	0.35	0.492	0.076

49	15-52F	0.300	0.030	0.0510	0.097	1.921	82000	2.1020	1.52	20.11	0.19	0.468	0.073
50	10-58F	0.235	0.055	0.0515	0.169	1.507	82000	2.1030	1.01	19.92	0.77	0.367	0.057
51	10-56F	0.225	0.047	0.0517	0.145	1.446	82500	2.1034	0.99	19.84	0.59	0.352	0.055
52	10-54F	0.216	0.036	0.0510	0.109	1.390	82900	2.1020	1.01	20.11	0.36	0.339	0.053
53	10-52F	0.205	0.025	0.0509	0.079	1.315	83400	2.1018	1.01	20.15	0.19	0.321	0.050
54	25-28F	0.320	0.040	0.0209	0.122	2.048	83400	2.0418	2.48	48.35	0.73	0.507	0.079
55	25-26F	0.310	0.030	0.0209	0.105	2.010	82800	2.0418	2.48	48.35	0.49	0.497	0.077
56	25-24F	0.307	0.029	0.0200	0.086	1.967	82300	2.0400	2.51	50.50	0.41	0.487	0.076
57	25-22F	0.298	0.019	0.0205	0.060	1.910	81700	2.0410	2.49	49.28	0.19	0.473	0.074
58	20-28F	0.260	0.036	0.0193	0.112	1.665	81700	2.0386	2.06	52.31	0.80	0.412	0.064
59	20-26F	0.253	0.030	0.0193	0.097	1.626	81300	2.0386	2.05	52.31	0.60	0.403	0.063
60	20-24F	0.247	0.026	0.0199	0.077	1.588	80800	2.0398	2.00	50.75	0.41	0.393	0.061
61	20-22F	0.241	0.018	0.0197	0.054	1.538	80400	2.0394	2.03	51.26	0.20	0.381	0.060
62	15-28F	0.199	0.033	0.0207	0.097	1.278	80400	2.0414	1.48	48.81	0.78	0.316	0.049
63	15-26F	0.196	0.028	0.0204	0.081	1.255	81100	2.0408	1.50	49.52	0.57	0.311	0.049
64	15-24F	0.190	0.023	0.0208	0.067	1.217	81700	2.0416	1.48	48.58	0.39	0.301	0.047
65	15-22F	0.183	0.016	0.0203	0.049	1.173	82400	2.0406	1.50	49.76	0.21	0.290	0.045
66	10-28F	0.139	0.028	0.0213	0.079	0.889	82400	2.0426	0.97	47.45	0.75	0.220	0.034
67	10-26F	0.137	0.027	0.0202	0.068	0.861	82300	2.0404	0.99	50.00	0.66	0.213	0.034
68	10-24F	0.130	0.019	0.0199	0.057	0.832	82300	2.0398	1.01	50.75	0.42	0.206	0.032
69	10-22F	0.125	0.015	0.0200	0.039	0.795	82200	2.0400	0.99	50.50	0.23	0.197	0.031

Table II gives the results of the tests and the DAPS4 calculations. The third column in Table II gives the actual observed collapse pressure from the tests. The fourth column gives the observed mode of collapse reported, where A = axisymmetric collapse, IGI = inelastic general instability, IIB = inelastic interbay buckling, and EGI = elastic general instability. The fifth column gives the DAPS4 computed pressure for axisymmetric collapse, P_A. The seventh column gives the DAPS4 computed pressure for general instability, P_{GI}. The ninth column gives the DAPS4 computed pressure for interbay buckling, P_{IB}. The sixth, eighth, and 10th columns give the ratio of the test collapse pressure to the respective DAPS4 computed pressure in the previous columns. The entries given in bold in the sixth, eighth, and 10th columns indicate the correlations of the individual theories with the actual observed failure modes. Numbers greater than unity indicate a conservative collapse pressure prediction because the shell collapsed at a higher pressure than computed; conversely, correlations less than unity indicate an unconservative prediction.

In addition to DAPS4 correlations with the tests, Table II includes correlations of the tests with the BOSOR5 code done and reported by David Bushnell (Lockheed Palo Alto Research Laboratory), author of the BOSOR4 and BOSOR5 codes, in reference 9. The BOSOR5 (Buckling Of Shells Of Revolution, Version 5) code was written for the analysis of segmented and branched ring-stiffened shells of revolution of multi-material construction. The theories in BOSOR5 are based on finite difference energy minimization in which moderately large meridional rotations, elastic-plastic effects, and primary or secondary creep are included.

The last three columns in Table II were taken directly from reference 9. The column labeled $P_{adjusted}$ is approximately the observed collapse pressure multiplied by the ratio of the outer surface radius to the middle surface radius. This was done in recognition that the BOSOR5 discretized computer models used the middle surface as the shell reference surface and the pressure acts over a larger area, more significantly in the cases of the thicker shells (smaller r/t). This adjustment was not necessary with the DAPS4 results because this effect is included internally in the code, as noted in the equations in the previous sections where $\alpha = R_o/r$.

Table II. DAPS4 and BOSOR5 Correlations With the Boichot and Reynolds Tests

	Model	P_{test}	Mode	P_A	P_{test}/P_A	P_{GI}	P_{test}/P_{GI}	P_{IB}	P_{test}/P_{IB}	$P_{adjusted}$	BOSOR5	$P_{adjusted}$/ BOSOR5
1	25-88	9450	A	8750	**1.080**	10580	0.893	9207	1.026	9875	9170	1.077
2	25-86	9550	A	8576	**1.114**	9540	1.001	8810	1.084	9980	9310	1.072
3	25-84	8850	IGI	8324	1.063	8659	**1.022**	8898	0.995	9237	9400	0.983
4	25-82	7275	IGI	7942	0.916	7540	**0.965**	8578	0.848	7593	8400	0.904
5	20-88	10600	A	9283	**1.142**	11380	0.931	9960	1.064	11062	10080	1.097
6	20-86	9750	IGI	9020	1.081	9758	**0.999**	8960	1.088	10176	10240	0.994
7	20-84	8800	IGI	8735	1.007	8926	**0.986**	9388	0.937	9105	9900	0.920
8	20-82	7400	IGI	8295	0.892	7955	**0.930**	8675	0.853	7696	8490	0.906
9	15-88	11600	IGI	10006	1.159	10920	**1.062**	10520	1.103	12105	11760	1.029
10	15-86	10000	IGI	9594	1.042	10090	**0.991**	10880	0.919	10435	11280	0.925
11	15-84	9100	IGI	9134	0.996	9321	**0.976**	9810	0.928	9462	10080	0.939
12	15-82	7650	IGI	8515	0.898	8309	**0.921**	9246	0.827	7926	8660	0.915
13	10-88	12100	IGI	10468	1.156	11280	**1.073**	14330	0.844	12658	13000	0.974
14	10-86	10650	IGI	9954	1.070	10590	**1.006**	13130	0.811	11140	11500	0.969
15	10-84	9150	IGI	9348	0.979	9674	**0.946**	12520	0.731	9468	10120	0.936
16	10-82	7650	IGI	8614	0.888	8509	**0.899**	9420	0.812	7937	8750	0.907
17	25-58	5600	A	5694	**0.983**	6321	0.886	6134	0.913	5740	5725	1.003
18	25-56	5600	A	5549	**1.009**	5894	0.950	5770	0.971	5739	5750	0.998
19	25-54	5550	IGI	5437	1.021	5460	**1.016**	5610	0.989	5678	5820	0.976
20	25-52	4470	IGI	5197	0.860	4879	**0.916**	5246	0.852	4559	5200	0.877
21	20-58	6300	A	6163	**1.022**	6691	0.942	6493	0.970	6488	6375	1.018
22	20-56	6300	A	5962	**1.057**	6238	1.010	6156	1.023	6488	6440	1.007
23	20-54	5450	IGI	5665	0.962	5741	**0.949**	5744	0.949	5559	6150	0.904
24	20-52	4500	IGI	5302	0.849	4874	**0.923**	5305	0.848	4595	5260	0.874
25	15-58	7050	IGI	6406	1.101	6891	**1.023**	6602	1.068	7205	7080	1.018
26	15-56	6400	IGI	6202	1.032	6562	**0.975**	6312	1.014	6544	6950	0.942
27	15-54	5500	IGI	5861	0.938	5873	**0.936**	5910	0.931	5610	6240	0.899
28	15-52	4600	IGI	5425	0.848	5054	**0.910**	5447	0.845	4742	5320	0.891
29	10-58	7250	IGI	6770	1.071	7245	**1.001**	6973	1.040	7458	7875	0.947
30	10-56	6450	IGI	6396	1.008	6656	**0.969**	6562	0.983	6634	7080	0.937
31	10-54	5625	IGI	6049	0.930	6121	**0.919**	6215	0.905	5779	6280	0.920
32	10-52	4700	IGI	5547	0.847	5287	**0.889**	5724	0.821	4812	5340	0.901
33	25-28	1850	A?	2101	**0.881**	2298	0.805	2173	0.851	1868	1920	0.973
34	25-26	1960	A?	2171	**0.903**	2248	0.872	2170	0.903	1990	2080	0.957
35	25-24	1880	IGI	2297	0.818	2095	**0.897**	2121	0.886	1892	2120	0.892
36	20-28	2225	A	2524	**0.882**	2652	0.839	2464	0.903	2239	2340	0.957
37	20-26	2150	A	2462	**0.873**	2479	0.867	2356	0.913	2162	2380	0.908
38	20-24	2075	IGI	2410	0.861	2251	**0.922**	2268	0.915	2085	2360	0.883
39	15-28	2400	A	2676	**0.897**	2778	0.864	2543	0.944	2424	2620	0.925
40	15-26	2360	IGI	2583	0.914	2547	**0.927**	2436	0.969	2389	2560	0.933
41	15-24	2060	IGI	2425	0.849	2251	**0.915**	2281	0.903	2093	2350	0.891
42	10-28	2625	IGI	2810	0.934	2870	**0.915**	2702	0.972	2666	2800	0.952
43	10-26	2360	IGI	2655	0.889	2627	**0.898**	2555	0.924	2398	2590	0.926
44	10-24	2040	IGI	2454	0.831	2322	**0.879**	2364	0.863	2070	2340	0.885
45	10-22	1640	IGI	2242	0.731	1888	**0.869**	2175	0.754	1664	1920	0.867
46	15-58F	7400	A	6325	**1.170**	6787	1.090	6517	1.135	7584	7060	1.074

47	15-56F	6750	IGI	6019	1.121	6363	**1.061**	6125	1.102	6917	6800	1.017
48	15-54F	5850	IGI	5643	1.037	5591	**1.046**	5691	1.028	5999	5920	1.013
49	15-52F	4920	IGI	5257	0.936	4904	**1.003**	5285	0.931	5042	5140	0.981
50	10-58F	7700	IGI	6583	1.170	7018	**1.097**	6778	1.136	7897	7620	1.036
51	10-56F	6800	IGI	6335	1.073	6579	**1.034**	6509	1.045	7013	6940	1.011
52	10-54F	5950	IGI	5794	1.027	5825	**1.021**	5946	1.001	6105	5940	1.028
53	10-52F	5000	IGI	5380	0.929	5116	**0.977**	5543	0.902	5099	5140	0.992
54	25-28F	2160	A	2330	**0.927**	2482	0.870	2355	0.917	2172	2160	1.006
55	25-26F	2080	A	2260	**0.920**	2253	0.923	2216	0.939	2106	2180	0.966
56	25-24F	1935	IIB	2114	0.915	2021	0.957	2057	**0.941**	1947	2060	0.945
57	25-22F	1420	EGI	2061	0.689	1299	**1.093**	1939	0.732	1439	1600	0.899
58	20-28F	2135	A	2299	**0.929**	2437	0.876	2265	0.943	2164	2260	0.958
59	20-26F	2100	A	2222	**0.945**	2267	0.926	2143	0.980	2113	2140	0.987
60	20-24F	2060	IGI	2197	0.938	2091	**0.985**	2087	0.987	2086	2180	0.957
61	20-22F	1520	EGI	2017	0.754	1435	**1.059**	1891	0.804	1527	1700	0.898
62	15-28F	2580	A	2627	**0.982**	2722	0.948	2521	1.023	2606	2620	0.995
63	15-26F	2400	IGI	2473	0.970	2447	**0.981**	2349	1.022	2421	2460	0.984
64	15-24F	2180	IGI	2398	0.909	2245	**0.971**	2270	0.960	2210	2250	0.982
65	15-22F	1720	EGI	2180	0.789	1721	**0.999**	2054	0.837	1729	1840	0.940
66	10-28F	2840	IGI?	2853	0.995	2910	**0.976**	2758	1.030	2854	2680	1.065
67	10-26F	2540	IGI?	2551	0.996	2649	**0.959**	2543	0.999	2556	2560	0.998
68	10-24F	2180	IGI	2384	0.914	2274	**0.959**	2297	0.949	2194	2280	0.962
69	10-22F	1775	IGI	2193	0.809	1872	**0.948**	2125	0.835	1788	1880	0.951

Figure 10 plots the correlations of Table II. The DAPS4 results plotted are the ones given in bold in Table II, i.e., the plotted DAPS4 result is the computed value for the actual mode of collapse observed in the test. There is very good consistency with the results from the BOSOR5 code. Also notable was the distinction between the correlations (for both DAPS4 and BOSOR5) with the test specimens with fillets versus those without fillets; the predictions tend to be more conservative overall for those specimens with fillets in comparison to those without fillets. It is not the purpose of this report to delve into the mechanisms that initiated the buckling of these shells; references 8 and 9 covered that discussion thoroughly already. However, it is instructive to include an excerpt from reference 9 on this discussion:

"… it appears that practically all of the specimens without fillets fractured during failure. … It is not possible to determine from the test data alone whether fracture caused the failure or whether fracture occurred later as the shell was deforming in its buckling mode. On the other hand, there is almost no evidence of fracture occurring in the case of the 24 specimens with fillets. Therefore, it is reasonable to predict that better agreement between test and theory will be obtained for the specimens with fillets than those without. Furthermore, analytical predictions that are too high for the specimens without fillets would lead one to favor the hypothesis that failure was caused by fracture rather than buckling in these tests, since the analytical model is not capable of predicting fracture. This would be particularly true if the too high predictions correspond to the thicker specimens for which imperfections are less significant."

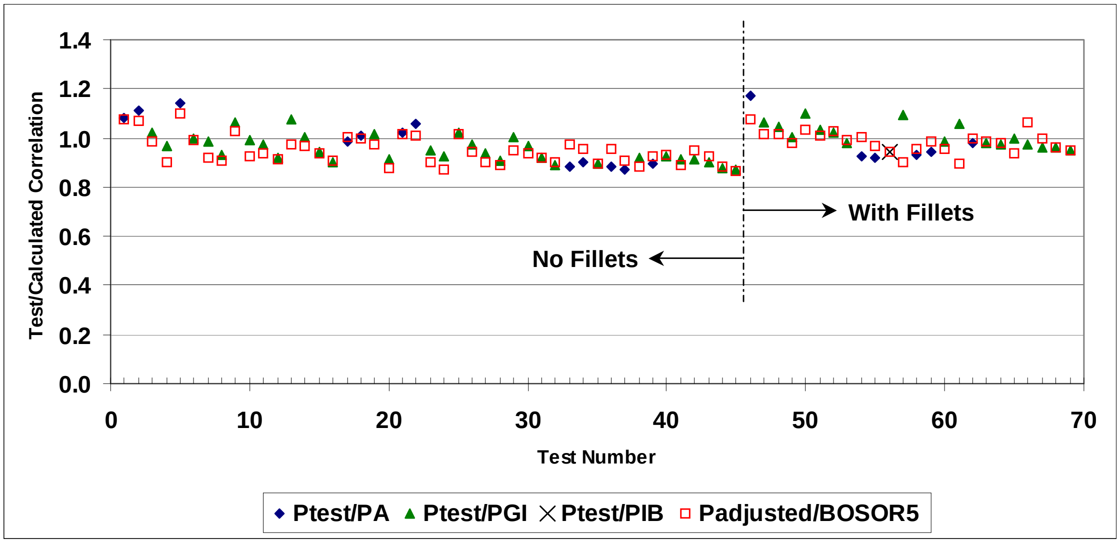


Figure 10. Correlations of Tests With DAPS4 (Actual Collapse Modes) and BOSOR5

Unfortunately, only one of the 69 tests failed in the inelastic interbay buckling failure mode, model number 56 (25-24F), so multiple data points are not available for this collapse mode from this data set. However, as seen in Figure 10, the correlation factors for this test were nearly identical for DAPS4 and BOSOR5 at 0.941 and 0.945, respectively, and are fairly good results.

Figure 11 was created using the minimum collapse pressure computed by DAPS4, regardless of the actual mode of collapse observed in the test. As a result, a number of the DAPS4 correlation points plotted are more conservative than shown in Figure 10. Figure 10 above is more relevant for the purpose of comparing the individual theories with tests that correspond to the actual collapse mode; however, Figure 11 is more relevant from a design point of view because one would normally use the minimum pressure collapse mode calculated as the critical buckling pressure.

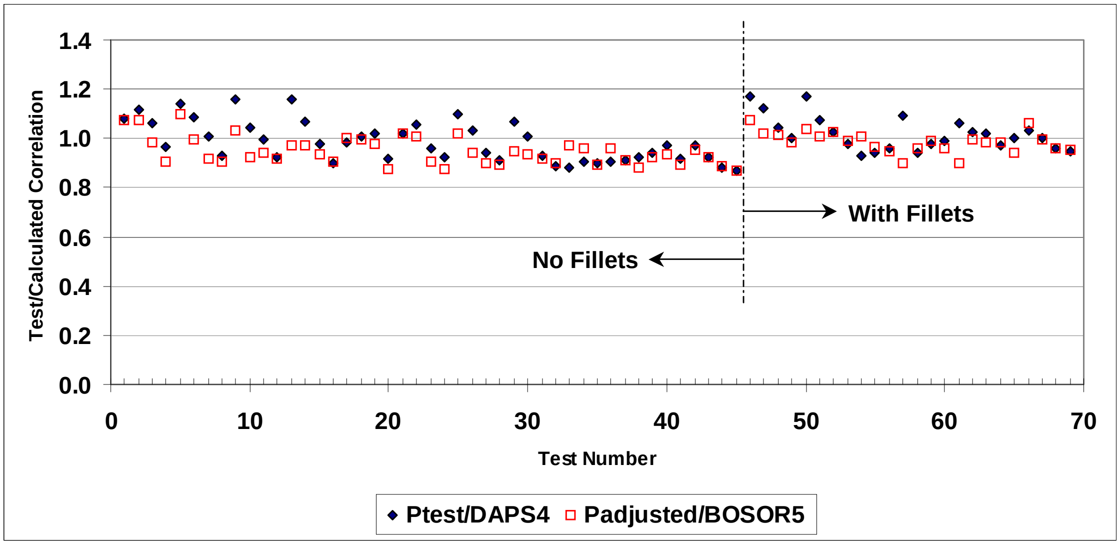


Figure 11. Correlations of Tests With DAPS4 (Minimum Collapse Pressures) and BOSOR5

Figure 12 plots the correlations with test as a function of L/r (on the left) for those shells that failed by general instability or a function of L_f/r (on the right) for those shells that failed by axisymmetric collapse. Overall, better agreement was with the shells with fillets. Otherwise, the DAPS4 correlations are rather insensitive to these dimensionless parameters.

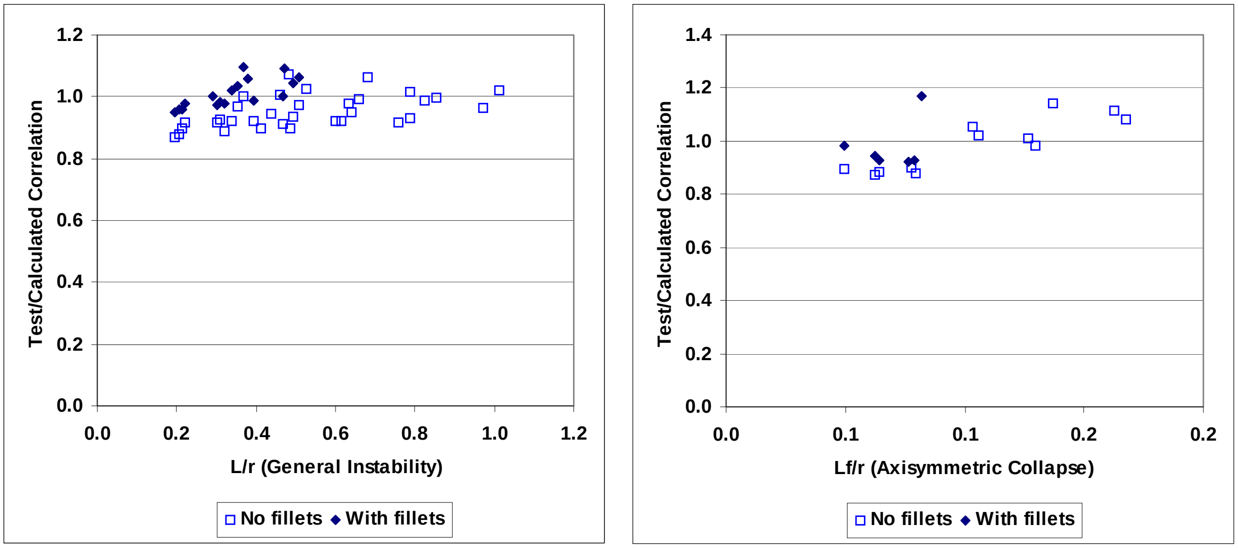


Figure 12. Correlations of Tests With DAPS4 vs. L/r or L_f/r

Figure 13 plots the correlations with test as a function of θ, r/t, and $A_f/(tL_f)$ and distinguish those that failed by axisymmetric collapse and general instability. The DAPS4 predictions are relatively insensitive to these three dimensionless parameters. It is noteworthy that all of the tests of the thickest shells (r/t ~ 12) failed by axisymmetric collapse and the thinner shells (r/t ~ 20 or 50) failed by general instability. Test 56 (25-24F) with r/t = 50.5 (not included in Figure 12) failed by inelastic interbay buckling.

The Lunchick plastic reserve strength factor for the axisymmetric collapse calculations was included in all cases. For the 18 cases in which the test models failed by axisymmetric collapse, the plastic reserve strength factor ranged from 1.064 to 1.183 with an average value of 1.113. Reviewing these 18 cases on an individual basis showed that the correlations clearly benefited from inclusion of this factor in the calculations, though it is optional in the input whether to include this effect.

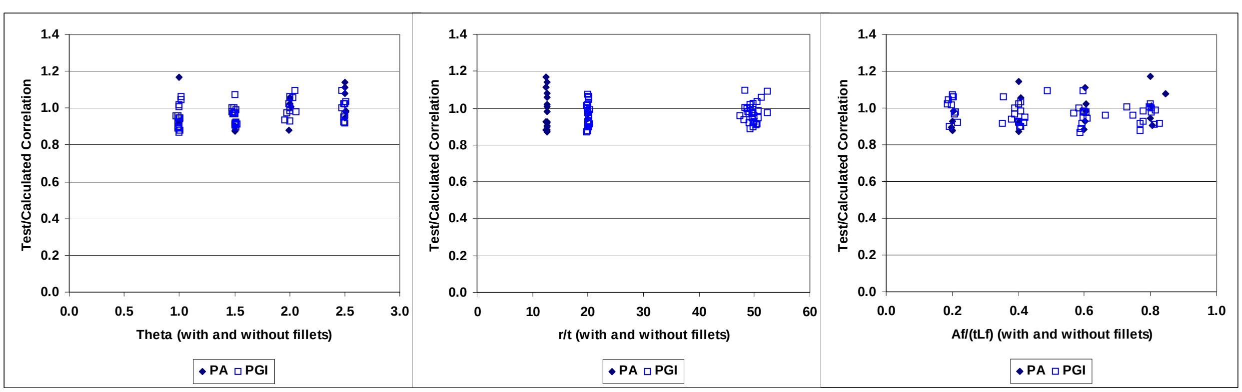


Figure 13. Correlations of Tests With DAPS4 Versus Other Dimensionless Parameters

USER MANUAL

Program Options

1.a When the elastic buckling pressure, PEL, has been calculated by some source other than DAPS4, (e.g. BOSOR4), of any shell of revolution made of an isotropic material, this option will calculate the inelastic pressure by using the tangent modulus vs. stress curve of the material. A linear stress analysis must also have been run at some pressure PSTR.

1.b Same as option 1.a except the shell material is orthotropic.

2.a This option will analyze (stress and stability) an isotropic monocoque cylindrical shell. The end ring properties may be input. (This option might be useful to analyze one bay of a ring-stiffened shell.)

2.b Same as option 2.a except the shell material is orthotropic. Note that the end rings may be a different material than the skin.

2.c This option will analyze (stress and stability) an isotropic ring-stiffened cylindrical shell of constant ring size and spacing. The rings may be internal or external. Limited to symmetric ring cross section (i.e., product of inertia = 0.).

2.d Same as option 2.c except the shell material is orthotropic.

2.e This option only does a stress analysis (no buckling) of any shell of options 2.a through 2.d (isotropic or orthotropic) under an internal pressure. (PSTR is negative.)

3.a This option will design an isotropic monocoque shell. End rings are not designed but end ring properties may be input as in option 2.a.

3.b Same as option 3.a except the shell material is orthotropic. Note that the end rings may be a different material than the skin.

3.c This option will design an isotropic ring-stiffened cylindrical shell of constant size and spacing. Limited to rectangular rings. The rings may be internal or external. The program iterates on the ring size and shell thickness until either the axisymmetric collapse or interbay buckling pressure as well as the general instability pressure have converged on the design pressure, PSTR. Therefore, two out of the three types of collapse considered will be at PSTR and the third will be ≥ PSTR.

3.d Same as option 3.c except the shell material is orthotropic.

Input Detailed Description

Mandatory line:

Line 1: `NPLOT,NCURVE,NDESIGN,ORTHO,NPRNT,INTERN,CONSRV,KMAX,NMAX`

Set `NPLOT` = 1 for no plots. All former plotting options have been disabled.

`NCURVE` must be 1, 2, 3, or 4. The four options are:
`NCURVE` = 1 for an isotropic shell. Lines 12, 13, and 14 will be read in to describe the isotropic compressive stress-strain curve.
`NCURVE` = 2 for a specially orthotropic shell. Lines 12 through 16 will be read in. The frame is assumed to have the same curve as the shell hoop curve. (Use `NCURVE` = 2 for option 1.b.)
`NCURVE` = 3 for an orthotropic ring-stiffened shell. Same as `NCURVE` = 2 except also read in Lines 17 and 18 to describe the frame material compressive stress-strain curve. The most general orthotropic, hybrid shell is the case `ORTHO` = 1 and `NCURVE` = 3. A special hybrid shell can be one in which the skin is isotropic and the frame is another material. In this case let `NCURVE` = 3 and `ORTHO` = 0 and input identical X and Y curves.
`NCURVE` = 4 when no stress-strain curves at all are used and only the elastic values read in will be used. You must use `NCURVE` = 4 for `PSTR` negative, program option 2.e or the special `NDESIGN` = -1 option. You may use `NCURVE` = 4 for any of the options 2.a through
3.d. Yield stress value(s) (Line 6) are required even when `NCURVE` = 4.

`NDESIGN` must be –1, 0, 1, or 2. The four options are:
`NDESIGN` = 0 is an option to analyze a ring-stiffened or monocoque shell (program options 1.a through 2.e).
`NDESIGN` = 1 is an option to design a ring-stiffened or monocoque shell (program options 3.a through 3.d.).
`NDESIGN` = 2 is an option to search for the optimum number of rings. Elaboration on the input requirements is given in the input description for Lines 10 and 11.
`NDESIGN` = -1 is a special case of program options 2.b, 2.d, and 2.e (analysis of orthotropic shells, no design) where the extensional (`A11` and `A22`) and flexural (`D11` and `D22`) rigidity terms are input (Line 3). No curves are read in since no plastic effects can be included, i.e., set `NCURVE` = 4. (`EX`, `EY`, and `VPLAS` are set to zero. `VYX`, `GXY`, and the yield stress values `SIGX`, `SIGY`, `SIGX1`, and `SIGY1` are still needed. Note that `GXY` = A_{66}/t or `GXY` = 12*D_{66}/t**3 and A_{12} = `VXY*A11` = `VYX*A22` and D_{12} = `VXY*D11` = `VYX*D22`.)

`ORTHO` = 0 for an isotropic shell (options 1.a, 2.a, 2.c, 2.e, 3.a, and 3.c). `ORTHO` = 1 for an orthotropic shell (options 1.b, 2.b, 2.d, 2.e, 3.b, and 3.d). In general, an orthotropic hybrid shell in DAPS4 can have different properties in the axial (X) and hoop (Y) directions. The rings can be another material, isotropic or unidirectional orthotropic. For a hybrid shell composed of an isotropic skin and a different frame material, set `ORTHO` = 0 and `NCURVE` = 3.

NPRNT = 0 for regular output. NPRNT = 1 for full output (recommended).

Set INTERN = 0 to design a shell with internal stiffeners. Set INTERN = 1 to design a shell with external stiffeners. INTERN is also used in options 2.a through 3.b when AR = -1. E.g., for the analyses of the externally stiffened shells of reference 8 (discussed in the previous section), INTERN = 1 was used. Also, for a monocoque shell, the end rings may be internal or external.

CONSRV = 0 (recommended) will cause DAPS4 to utilize Lunchick's theory for including the plastic reserve strength for axisymmetric collapse in the analysis or design process. Set CONSRV = 1 to prevent this. Whether CONSRV is 0 or 1, DAPS4 will print the plastic reserve strength factor for your information (options 2.a, 2.c, 3.a, and 3.c). CONSRV = 0 is an option only when ORTHO = 0 (an isotropic shell).

KMAX = the number of terms in the series defining the assumed displacement shape for interbay or monocoque shell buckling. The maximum number of half-waves used in the solution will be (KMAX - 1)*2 + 1. For example, if KMAX = 4, the solution includes 1, 3, 5, and 7 half-waves per bay. (Only an odd number of half-waves per bay are used.) The energy matrix is of order (3*KMAX) by (3*KMAX). Default value is KMAX = 4. Maximum value for KMAX allowed is 8 (recommended value).

NMAX = the maximum mode considered for interbay (or monocoque shell) buckling. If the minimum mode computed by DAPS4 turns out to be NMAX, then you may not have found the minimum and should increase NMAX accordingly. Default value is 18 (recommended value). (Note that NMAX for general instability calculations is defaulted at 12, which should be sufficient.)

Mandatory line:

Line 2: TITLE

TITLE = up to 70 character string. This information appears on the printed output.

If (NDESIGN.EQ.-1), read:

Line 3: A11,A22,D11,D22

See the description for Line 1, NDESIGN = -1.

Mandatory line:

Line 4: EX,EY,EF,GXY,GF,VYX,VPLAS,VF

EX = isotropic elastic modulus for ORTHO = 0 or axial elastic modulus for ORTHO = 1.

EY = hoop modulus for ORTHO = 1. For ORTHO = 0, set to zero.

EF = ring hoop modulus for ORTHO = 1. For (ORTHO = 0 .AND. NCURVE = 1), set to zero.

GXY = the in-plane shear modulus for the hull material. For ORTHO = 0, set to zero and the isotropic shear modulus will be used.

GF = ring shear modulus for torsion when ORTHO = 1. For (ORTHO = 0 .AND. NCURVE = 1), set to zero; however, EF, GF, and VF must be read in when (ORTHO = 0 .AND. NCURVE = 3). Also, if (NCURVE = 4 .AND. ORTHO = 0), EF, GF, VF, and RHOF may be read in with properties different from the skin.

VYX = strain in X-direction over strain in Y-direction due to stress in Y-direction. Note: if the material longitudinal direction coincides with the shell axial direction, then VYX is the minor Poisson ratio (EX*VYX = EY*VXY). Also, when NCURVE = 1, 2, or 3, DAPS4 imposes theoretical maximums on the elastic and plastic Poisson ratios.

VPLAS is the value of VYX at yield (SIGX) and must not be zero. A linear variation of VYX (and subsequently VXY) is automatically calculated for use in the stability routines to be consistent with the calculated tangent moduli. Beyond SIGY, VYX is held constant at VPLAS and VXY still changes with the moduli to be consistent. (The stress routines always use the elastic moduli and Poisson ratios.) For isotropic materials, the following relationship may be useful for computing VPLAS if test data are not available: VPLAS = (VYX *SIGX + 0.001*EX) / (SIGX + 0.002*EX). Note that when ORTHO = 0, SIGX = SIGY and VXY = VYX.

VF = ring Poisson ratio. For (ORTHO = 0 .AND. NCURVE = 1), set to zero.

Mandatory line:

Line 5: `PSTR,PEL,STRX,STRY,NSAVE`

PSTR = stress analysis pressure of program options 1.a through 2.e. PSTR = the design pressure of options 3.a through 3.d. PSTR is positive in all but option 2.e, units of psi.

PEL = the elastic buckling pressure (positive, units of psi) of option 1.a or 1.b. PEL must = 0. for all other options.

STRX = the mid-bay mid-plane Von Mises stress of the critical bay due to PSTR in option 1.a. You must calculate the Von Mises stress from the mid-plane hoop and axial stresses. STRX = the axial mid-bay mid-plane stress (positive, units of psi) of the critical bay due to PSTR in option 1.b. Set STRX to zero for all options other than 1.a or 1.b.

STRY = the circumferential mid-bay mid-plane stress (positive, units of psi) of the critical bay due to PSTR in option 1.b (ORTHO = 1). Set STRY to zero for all other options.

NSAVE = the hoop wave number of option 1.a or 1.b. and is not used other than in the printed output. Set to zero for all other options.

Mandatory line:

Line 6: SIGX,SIGY,SIGX1,SIGY1

SIGX = axial (or isotropic) compressive 0.002 offset yield stress, units of psi. This must be input as a positive number. See the description of SIGXNOM and SIGYNOM below (Line 12) for the significance of reading in SIGX and/or SIGY different from SIGXNOM and/or SIGYNOM, respectively.

SIGY = compressive yield in Y-direction (positive). Set to zero if ORTHO = 0.

SIGX1 = tensile yield in X-direction (positive). Set to zero if ORTHO = 0.

SIGY1 = tensile yield in Y-direction (positive). Set to zero if ORTHO = 0.

Mandatory line:

Line 7: L,OD,T,RHO,RHOF,ARM,ENDB

L = shell length (inches), not including any end bulkheads or heavy terminating rings.

OD = outside diameter of the shell plating.

T = skin thickness, needed for options 1.a through 2.e. Set to zero for design options 3.a through 3.d.

RHO = skin density (units of lb/in^3), needed for options 2.a through 3.d. Set to zero for options 1.a and 1.b.

RHOF = ring density, needed for ring-stiffened shells. Set to zero for a monocoque shell.

ARM = area of an end ring of a monocoque shell (options 2.a, 2.b, 3.a, and 3.b). ARM must = 0. for ring-stiffened shells (options 2.c, 2.d, 3.c, and 3.d).

ENDB = the width (axial dimension) of an end ring of a monocoque shell (options 2.a, 2.b, 3.a, and 3.b). Note that L does not include the ENDB dimension. Set to zero for a ring-stiffened shell.

If (PEL.LE.0.), read:

Line 8: AR,LF,LS,IR,IX,J,YR

AR = area (units of square inches) of one ring for options 2.c, 2.d, and 2.e (if ring-stiffened). AR must = 0. for all other options. A special feature for options 2.a through 3.b is provided whereby you must set AR = -1.0. Then the ring dimensions are input via Line 9. When AR = -1.0 and ARM = 0., AR, LS, IR, IX, J, and YR will be calculated by DAPS4 but LF is still required input. (When AR = -1. and ARM > 0., the monocoque shell end ring properties IR, IX, J, and YR are computed and ARM is recomputed using the input ring dimensions, but the input value of ENDB is still used.)

LF = the ring spacing, C.G. to C.G. (ring-stiffened shell). Set to zero for a monocoque shell.

LS = the unsupported bay length (= LF minus the ring width) of a ring-stiffened shell. Set to zero when AR = -1.

IR = inertia of a ring about an axis through its own C.G. (the axis being parallel to the shell axis of symmetry). Set to zero when AR = -1.

IX = inertia of a ring about a radial line through its own C.G. Set to zero when AR = -1.

J = ring torsion constant. Set to zero when AR = -1.

YR = distance from outer shell surface to C.G of a ring, positive inward. Set to zero when AR = -1.

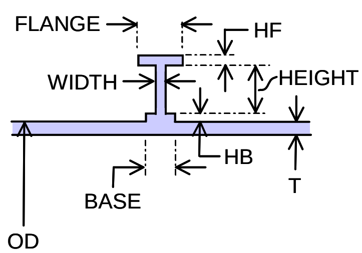
If (AR.EQ.-1.), read:

Line 9: BASE,HB,WIDTH,HEIGHT,FLANGE,HF

BASE = faying width of flange in contact with the shell, and
HB = thickness of this flange.

WIDTH = thickness of web, and
HEIGHT = height of web.

FLANGE = width of standing flange, and
HF = thickness of this flange.



Either or both flanges may be zero. If both flanges are zero, then the ring is rectangular with dimensions WIDTH x HEIGHT. Do not allow the standing flange to be the only non-zero component. All of the following must be true or J will not be correctly calculated: (BASE $\geq$ HB), (HEIGHT $\geq$ WIDTH), and (FLANGE $\geq$ HF). For example, if the ring is rectangular and has an aspect ratio less than 1.0, use BASE and HB to define it; if the aspect ratio is greater than 1.0, then use WIDTH and HEIGHT to define it. Note that INTERN (Line 1) will control whether the ring is internal or external.

If (NDESIGN.GT.0), read:

Line 10: RINGS,RINGSI,RINGSF,DRINGS

RINGS is the number of rings to be designed in the design option, not including end rings. For example, there are six rings in Figure 9. To design a monocoque shell, set RINGS = 0.

There is an option to repeat the shell design in a do-loop sense where only the number of rings changes. The range is from RINGSI to RINGSF incremented by DRINGS, where RINGS must be equal to RINGSI. This internal stacking gives all the output as if the data actually were stacked. If no internal ring stacking is desired, leave RINGSI, RINGSF, and DRINGS zero.

A search option (NDESIGN = 2) has the program search for the optimum number of rings (by weight). Weights and trial numbers of rings are tabulated. The search range will be RINGSI to RINGSF unless RINGSI and RINGSF are zero in which case the program will use default values; RINGS and DRINGS are ignored with NDESIGN = 2.

If (NDESIGN.GT.0), read:

Line 11: SECDEP,DW,DASP,DWMIN,DASPMAX,ISEC

The design option is limited to rectangular rings.

SECDEP = the approximate section depth the shell is to have, ring height plus shell thickness.

DW = the maximum width the rings are allowed to have in the design.

DASP = the minimum aspect ratio (HEIGHT / WIDTH) of the ribs allowed in the design.

DWMIN and DASPMAX are used in combination: the final ring dimensions will satisfy the condition that if the aspect ratio is greater than DASPMAX, then the ring width will be ≥ DWMIN. Default values are DWMIN = 0.125 and DASPMAX = 4. SECDEP, DW, DASP, DWMIN, and DASPMAX are optional but it is highly recommended that trial values be used to help DAPS4 in the design iterations.

ISEC = 0 if SECDEP is allowed to be overridden in the design. If the designed shell must not have a section depth greater than SECDEP, let ISEC = 1. DASP still takes precedence, however.

If (NCURVE.EQ.4), end of input; otherwise input Lines 12, 13, and 14:

Line 12: SIGXNOM,SIGYNOM,NXPNT,NYPNT,NFPNT

SIGXNOM = the nominal 0.002 yield stress of the axial material stress-strain curve (or isotropic curve for ORTHO = 0). That is, SIGXNOM is the yield value on the curve actually read in. It is needed for curve options 1, 2, and 3 (positive value).

SIGYNOM = nominal hoop 0.002 yield stress of the circumferential material stress-strain curve (needed when ORTHO = 1).

Note if SIGX and SIGY are different from SIGXNOM and SIGYNOM, respectively, then the program internally shifts the curves by a ratio amount when computing the tangent moduli. The new curve has the same slope in the elastic region.

If NCURVE = 1, read in NXPNT and set NYPNT = NFPNT = 0. If NCURVE = 2, read in NXPNT and NYPNT and set NFPNT = 0. If NCURVE = 3, read in NXPNT, NYPNT, and NFPNT.

Line 13: (STRESSX(I), I = 1,NXPNT)

Read in NXPNT values of stress on the engineering compressive stress-strain curve, all positive values, and not including the origin. The first point should be the proportional limit. A maximum of 10 values are allowed. Units are psi and in/in. If ORTHO = 0, this is for an isotropic material. If ORTHO = 1, then this curve represents the hull material behavior in the axial direction.

Line 14: (STRAINX(I), I = 1,NXPNT)

Read in NXPNT values of strain on the engineering compressive stress-strain curve, all positive values, corresponding to the stress values of Line 13.

If (NCURVE.EQ.1), <u>end of input</u>; otherwise input Lines 15 and 16:

Line 15: (STRESSY(I), I = 1,NYPNT)

Read in NYPNT values of stress on the engineering compressive stress-strain curve, all positive values, and not including the origin. The first point should be the proportional limit. A maximum of 10 values are allowed. If ORTHO = 1, this curve represents the hull material behavior in the circumferential direction. If ORTHO = 0 (e.g., the special option of ORTHO = 0 and NCURVE = 3) then this is for an isotropic material and must be identical to Line 13.

Line 16: (STRAINY(I), I = 1,NYPNT)

Read in NYPNT values of strain on the engineering compressive stress-strain curve, all positive values, corresponding to the stress values of Line 15.

If (NCURVE.EQ.2), <u>end of input</u>; otherwise input Lines 17 and 18:

Line 17: (STRESSF(I), I = 1,NFPNT)

Read in NFPNT values of stress on the engineering compressive stress-strain curve, all positive values, and not including the origin. The first point should be the proportional limit. A maximum of 10 values are allowed. This curve represents the ring frame material behavior in the circumferential direction.

Line 18: (STRAINF(I), I = 1,NFPNT)

Read in NFPNT values of strain on the engineering compressive stress-strain curve, all positive values, corresponding to the stress values of Line 17.

Notes:

1. For the design option, it is highly recommended that the controls SECDEP, DW, DASP, DWMIN, DASPMAX, and ISEC be used. If difficulties are encountered with achieving convergence on good designs, recommend varying these parameters to learn their influence. The hierarchy of importance DAPS4 uses to size the ring is as follows:
a. DASP, the ratio HEIGHT / WIDTH will be ≥ DASP.

b. SECDEP, if ISEC = 1 and if DASP is satisfied, then the program will force the ring height plus shell thickness to equal SECDEP unless either (1) the aspect ratio is greater than DASPMAX and the ring width is less than DWMIN or (2) if the ring width reaches 1/3 of the ring spacing.
c. DW, if DW is greater than zero, DASP is satisfied, and ISEC = 0, then the maximum width of a ring allowed will be DW.

2. DAPS4 does not calculate ring crippling. Item b. above regarding the maximum aspect ratio and minimum ring width allows the designer to guard against ring crippling. (See definition of DWMIN and DASPMAX.) For isotropic shells approx 21 inches OD and smaller, the default values for DWMIN and DASPMAX should suffice. Otherwise, recommend checking with ABAQUS, PANDA, or BOSOR4.

3. Most input variables follow the standard format convention based on the first letter of the name. The exceptions are ORTHO and CONSERV (Line 1) are integers and L, LF, LS, IR, IX, and J (Lines 7 and 8) are floating point.

4. It recommended that the user check the value of the "Ring-Shell Stiffness Factor" that is printed out after the "Energy Coefficient Matrix For Elastic Interbay/Monocoque Shell Buckling" (with NPRNT = 1). The Ring-Shell Stiffness Factor is calculated from $\dfrac{E_f I_x + G_f J}{\sqrt{G_{LT}\sqrt{E_x E_y}\,t^3 L_f}} \cdot \dfrac{E_f A_f}{\sqrt{E_x E_y}\,t L_f} \cdot \dfrac{L}{D}$. This factor is typically much less than unity and can be as high as 1 to 10. If much higher than this, examine the sensibility of the case being analyzed. For example, the DAPS4 code is not intended to model a shell with a massive ring. Typically, the dominant coefficient in the printed elastic mode shape for the interbay instability pressure is W(1). If this is not the case and if the elastic interbay instability pressure is extremely greater than the DAPS3-type calculation (also printed out for your information), then the design should be considered bad and, preferably, be analyzed with another code such as ABAQUS and model the rings with solid elements with appropriate shell-to-solid coupling for connection of solid elements to shell elements. If using the design option in DAPS4 and this situation arises, refer back to the advice given in note number 1 above and adjust the ring parameters and/or the number of rings in the design.

5. Historical note: The program option 1.a is a carryover from the original predecessor of the DAPS series of codes called INELAS written by this author in 1976. In those days, codes for the calculation of inelastic collapse of shells were of limited capability and at best required expensive run times on the computer. Of course, this has changed with advanced code capabilities such as ABAQUS and modern day computers.

6. The data sets may be stacked.

Input Summary

Mandatory line:
Line 1: `NPLOT,NCURVE,NDESIGN,ORTHO,NPRNT,INTERN,CONSRV,KMAX,NMAX`
Mandatory line:
Line 2: `TITLE`
If (`NDESIGN.EQ.-1`), read:
Line 3: `A11,A22,D11,D22`
Mandatory line:
Line 4: `EX,EY,EF,GXY,GF,VYX,VPLAS,VF`
Mandatory line:
Line 5: `PSTR,PEL,STRX,STRY,NSAVE`
Mandatory line:
Line 6: `SIGX,SIGY,SIGX1,SIGY1`
Mandatory line:
Line 7: `L,OD,T,RHO,RHOF,ARM,ENDB`
If (`PEL.LE.0.`), read:
Line 8: `AR,LF,LS,IR,IX,J,YR`
If (`AR.EQ.-1.`), read:
Line 9: `BASE,HB,WIDTH,HEIGHT,FLANGE,HF`
If (`NDESIGN.GT.0`), read:
Line 10: `RINGS,RINGSI,RINGSF,DRINGS`
If (`NDESIGN.GT.0`), read:
Line 11: `SECDEP,DW,DASP,DWMIN,DASPMAX,ISEC`
If (`NCURVE.EQ.4`), end of input; otherwise input the next three lines:
Line 12: `SIGXNOM,SIGYNOM,NXPNT,NYPNT,NFPNT`
Line 13: (`STRESSX(I), I = 1,NXPNT`)
Line 14: (`STRAINX(I), I = 1,NXPNT`)
If (`NCURVE.EQ.1`), end of input; otherwise input the next two lines:
Line 15: (`STRESSY(I), I = 1,NYPNT`)
Line 16: (`STRAINY(I), I = 1,NYPNT`)
If (`NCURVE.EQ.2`), end of input; otherwise input the next two lines:
Line 17: (`STRESSF(I), I = 1,NFPNT`)
Line 18: (`STRAINF(I), I = 1,NFPNT`)

Example Problems

Appendix A gives a complete listing of the data sets used to analyze the Boichot and Reynolds tests of reference 8. Those cases are examples of program option 2.c, have external rectangular stiffeners, and the ring dimensions are input (`ORTHO` = 0, `INTERN` = 1, `AR` = -1.0). Appendix A also explains proper usage of the code to analyze the different failure modes in a shell that has uneven ring spacing. The following is a set of input decks for four DAPS4 runs to illustrate a few of the code's options. Table III gives some of the pertinent results of these runs.

Case 1. The following input is for an elastic analysis of a monocoque shell with a nominal end ring with a 1.0 in^2 cross section:

```
1  4  0  0  1  0  0  8  16
Analysis of an Elastic Monocoque Isotropic 6061-T6 Shell
10.0D+6   0.  0.  0.  0.   0.3  0.  0.3
560.  0.  0.  0.  0
41000.  0.  0.  0.
53.4  10.  0.250  .098  0.098  1.  1.
-1.  0.  0.  0.  0.  0.  0.
0.   0.   1.0   1.0   0.  0.
```

Case 2. The following input is for an analysis of a steel ring-stiffened shell with internal Tee-stiffeners:

```
1  1  0  0  1  0  0  8  0
Analysis of a Sch. 10 pipe with small T-rings; A53 Type S, Grade B
30.E+6  0.  0.  0.  0.  0.29  0.42  0.
1000.  0.  0.  0.  0
35000.  0.  0.  0.
11.  20.  0.250  .283  0.  0.  0.
-1.  2.75  0.  0.  0.  0.  0.
0.  0.  0.1  0.4  0.3  0.1
35000. 0. 9 0 0
25968.  31000.  33000.  35000.  37000.  38000.  45000.  51000.  60000.
0.00087  0.0011  0.0017  0.00317  0.0050  0.0060  0.0140  0.0210  0.30
```

Case 3. The following input is for the design of a large steel externally ring-stiffened shell:

```
1  1  1  0  1  1  0  8  20
Design of a Steel Ring-Stiffened Shell
30.E+6  0.  0.  0.  0.  .3  0.36  0.
1800.  0.  0.  0.  0
142000.  0.  0.  0.
300.   350.  0.  .285  0.  0.  0.
0.  0.  0.  0.  0.  0.  0.
10.  10.  10.  0.
21.  2.  21.  1.1  15.  0
130000. 0. 10 0 0
107100. 113000. 115000. 117500. 122500. 124000. 125500. 130000. 132500. 135000.
.003570 .003800 .003900 .004050 .004450 .004633 .004900 .006333 .010000 .020000
```

Case 4. The following input is for the design of an externally ring-stiffened shell using fake orthotropic properties to demonstrate the `ORTHO` = 1 and `NCURVE` =3 options. Titanium is input for the axial properties, HY-130 is input for the circumferential properties, and aluminum is input for the ring properties. In addition, designs are requested with four rings and nine rings using `RINGSI`, `RINGSF`, and `DRINGS`.

```
1  3  1  1  1  1  0  8  18
Fake Composite: Ti-5Al-2.5Sn (Axial); HY-130 (Hoop); 6061-T6 (Rings)
16.2+6  30.E+6  10.0D+6   7.E+6  3.85E+6   0.26  0.4  0.3
1500.  0.  0.  0.  0
120000.  125000.   135000.  132000.
100.  21.  0.  .2  0.098  0.  0.
0.  0.  0.  0.  0.  0.  0.
4.  4.  9.  5.
1.  .2   4.  .1  8.  1
131100.  130000.  8  10  9
120000.0  123900.0  125800.0  127600.0  128800.0  130200.0  131300.0  132900.0
0.0074  0.008  0.0084  0.0089  0.0093  0.01  0.0107  0.012
107100. 113000. 115000. 117500. 122500. 124000. 125500. 130000. 132500. 135000.
.003570 .003800 .003900 .004050 .004450 .004633 .004900 .006333 .010000 .020000
25333. 30000.  34950.  36533. 38067.  39433.  40433. 41033. 41667.
.0024733 .003 .0036667 .004 .0043333 .0048333 .0053333 .006 .0072
```

Table III. Selected Output From Four Example Cases

Case Number	Number of Rings	Axisymmetric Collapse	LPR	General Instability		Interbay Buckling		RSSF
				Elastic	Inelastic	Elastic	Inelastic	
1	0.	2427.	1.000	NA	NA	460.1	NA	0.3270
2	3.	1096.	1.016	6223.	1091.	15,920.	1221.	7.049E-04
3	10.	1931.	1.041	3280.	1798.	3316.	1801.	5.383E-03
4a	4.	3436.	NA	1501.	1501.	1500.	1500.	1.663E-02
4b	9.	2513.	NA	1501.	1501.	1500.	1500.	0.1175

In Table III, LPR = the "Lunchick Plastic Reserve Factor" defined in equation (66) and RSSF = the "Ring-Shell Stiffness Factor" described in note 4 above. The following is a sample DAPS4 output, taken from Case 2 above.

```
1    Analysis of a Sch. 10 pipe with small T-rings; A53 Type S, Grade B

     ELASTIC MODULI
       EX =  30.00E+06   EY =  30.00E+06   EF =  30.00E+06
       GXY =  1.163E+07   GF =  1.163E+07
     POISSON RATIOS
       VXY = 0.2900   VYX = 0.2900   VPLAS = 0.420   VF = 0.290
     SHELL WEIGHT =    56.52 LB/FT ,  INTERNAL VOLUME =    2.0661 CU.FT./FT.
     (SKIN DENSITY =  0.2830       RING DENSITY =  0.2830)

     OUTSIDE DIAMETER =  20.000   LENGTH =   11.000   CRITICAL LENGTH =  3.83
     BAY THICKNESS = 0.250   RING SPACING =   2.750
     RING PROPERTIES
        IR =  1.630E-03   IX =  2.583E-04   J =  1.914E-04
        AREA =  7.000E-02   RADIUS TO C.G. =   9.443   (ARM =   0.00    )

     RING PARTS READ IN
        BASE =   0.000       WIDTH =   0.100        FLANGE =   0.300  (AXIAL)
        HB =   0.000         HEIGHT =   0.400       HF =   0.100      (RADIAL)
        SECTION DEPTH =   0.750

     ANALYSIS PRESSURE IS   1000. PSI.

     SHELL STRESSES AT MIDBAY
                                  OUTSIDE            INSIDE
          LONGITUDINAL (PSI)       -22510.           -17997.
          CIRCUMFERENTIAL (PSI)    -37185.           -35877.
             VON MISES (PSI)        32441.            31070.
             PERCENT YIELD  *         92.7              88.8
          RADIAL DEFLECTION (IN.)           -0.0101

     MIDBAY MEMBRANE STRESS LEVEL IS    31699. PSI ( 90.6 PERCENT YIELD).

     SHELL STRESSES AT THE FRAME
                                  OUTSIDE            INSIDE
          LONGITUDINAL (PSI)       -15637.           -24869.
          CIRCUMFERENTIAL (PSI)    -34451.           -37128.
             VON MISES (PSI)        29878.            32766.
             PERCENT YIELD  *         85.4              93.6
          RADIAL DEFLECTION (IN.)           -0.0098
          HOOP STRESS IN FRAME (PSI)        -29916.

     *  YIELD STRESSES  (USED WITH VON MISES YIELD CRITERION)
                        HOOP (C) =  35000. PSI        AXIAL (C) =  35000. PSI
                        HOOP (T) =  35000. PSI        AXIAL (T) =  35000. PSI
     STRESS-STRAIN DATA ECHO ...  (STRESS IS IN KSI FOR ECHO ONLY.)
     X  25.97  31.00  33.00  35.00  37.00  38.00  45.00  51.00  60.00
       .00087 .00110 .00170 .00317 .00500 .00600 .01400 .02100 .30000

     AXISYMMETRIC COLLAPSE PRESSURE =  1096. PSI.
      (THE PLASTIC RESERVE STRENGTH FACTOR  1.016 WAS UTILIZED.)

     ENERGY COEFFICIENT MATRIX FOR ELASTIC GENERAL INSTABILITY
```

```
A11 =  4.232E+08    A22 =  1.148E+09    A33 =  3.762E+07    A44 =  4.390E+09    A55 =  2.001E+08
A12 = -1.564E+08    A13 =  5.749E+06    A14 = -2.023E+08    A15 =  7.436E+06    A23 = -4.384E+07
A24 =  1.361E+09    A25 = -1.112E+08    A34 = -1.112E+08    A35 =  2.055E+07    A45 = -2.596E+08
B11 =  1.190E+03    B22 =   227.        B33 =  1.266E+03    B44 =  2.255E+03    B55 =  5.648E+03
B12 =   146.        B13 =  -24.4        B14 =   189.        B15 =  -31.5        B23 =   42.4
B24 =   45.6        B25 =   11.3        B34 =   11.3        B35 =  1.508E+03    B45 =   26.1
GAMFE =  2.525E-08    GAMSE =  0.829
THE PERFORMANCE INDEX IS  0.804

ELASTIC GENERAL INSTABILITY PRESSURE =  6.223E+03     MODE =  5
***  INELASTIC BUCKLING PRESSURE =  1.091E+03 PSI.
THE STRESS LEVEL IN THE SHELL =    98.8809 PERCENT YIELD.
ELASTIC MODE SHAPE:
A1= 4.827E-02 B1= 1.744E-01 C1= 9.835E-01 B2= 2.717E-03 C2= 1.503E-03

ENERGY COEFFICIENT MATRIX FOR ELASTIC INTERBAY/MONOCOQUE SHELL BUCKLING
A( 1, 1)=  1.740E+09  A( 1, 2)= -9.383E+08  A( 1, 3)=  2.000E+07  A( 1, 4)=  2.279E+06  A( 1, 5)=   0.00      A( 1, 6)= -7.335E+06
A( 1, 7)=  2.279E+06  A( 1, 8)=   0.00      A( 1, 9)= -1.222E+07  A( 1,10)=  2.279E+06  A( 1,11)=   0.00      A( 1,12)= -1.711E+07
A( 1,13)=  2.279E+06  A( 1,14)=   0.00      A( 1,15)= -2.200E+07  A( 1,16)=  2.279E+06  A( 1,17)=   0.00      A( 1,18)= -2.689E+07
A( 1,19)=  2.279E+06  A( 1,20)=   0.00      A( 1,21)= -3.178E+07  A( 1,22)=  2.279E+06  A( 1,23)=   0.00      A( 1,24)= -3.667E+07
A( 2, 1)= -9.383E+08  A( 2, 2)=  2.645E+09  A( 2, 3)= -1.296E+08  A( 2, 4)=   0.00      A( 2, 5)=   0.00      A( 2, 6)=   0.00
A( 2, 7)=   0.00      A( 2, 8)=   0.00      A( 2, 9)=   0.00      A( 2,10)=   0.00      A( 2,11)=   0.00      A( 2,12)=   0.00
A( 2,13)=   0.00      A( 2,14)=   0.00      A( 2,15)=   0.00      A( 2,16)=   0.00      A( 2,17)=   0.00      A( 2,18)=   0.00
A( 2,19)=   0.00      A( 2,20)=   0.00      A( 2,21)=   0.00      A( 2,22)=   0.00      A( 2,23)=   0.00      A( 2,24)=   0.00
A( 3, 1)=  2.000E+07  A( 3, 2)= -1.296E+08  A( 3, 3)=  8.659E+07  A( 3, 4)= -2.445E+06  A( 3, 5)=   0.00      A( 3, 6)=  2.667E+06
A( 3, 7)= -2.445E+06  A( 3, 8)=   0.00      A( 3, 9)=  4.445E+06  A( 3,10)= -2.445E+06  A( 3,11)=   0.00      A( 3,12)=  6.223E+06
A( 3,13)= -2.445E+06  A( 3,14)=   0.00      A( 3,15)=  8.001E+06  A( 3,16)= -2.445E+06  A( 3,17)=   0.00      A( 3,18)=  9.779E+06
A( 3,19)= -2.445E+06  A( 3,20)=   0.00      A( 3,21)=  1.156E+07  A( 3,22)= -2.445E+06  A( 3,23)=   0.00      A( 3,24)=  1.333E+07
A( 4, 1)=  2.279E+06  A( 4, 2)=   0.00      A( 4, 3)= -2.445E+06  A( 4, 4)=  9.034E+09  A( 4, 5)= -2.815E+09  A( 4, 6)=  6.000E+07
A( 4, 7)=  2.279E+06  A( 4, 8)=   0.00      A( 4, 9)= -1.222E+07  A( 4,10)=  2.279E+06  A( 4,11)=   0.00      A( 4,12)= -1.711E+07
A( 4,13)=  2.279E+06  A( 4,14)=   0.00      A( 4,15)= -2.200E+07  A( 4,16)=  2.279E+06  A( 4,17)=   0.00      A( 4,18)= -2.689E+07
A( 4,19)=  2.279E+06  A( 4,20)=   0.00      A( 4,21)= -3.178E+07  A( 4,22)=  2.279E+06  A( 4,23)=   0.00      A( 4,24)= -3.667E+07
A( 5, 1)=   0.00      A( 5, 2)=   0.00      A( 5, 3)=   0.00      A( 5, 4)= -2.815E+09  A( 5, 5)=  5.234E+09  A( 5, 6)= -1.346E+08
A( 5, 7)=   0.00      A( 5, 8)=   0.00      A( 5, 9)=   0.00      A( 5,10)=   0.00      A( 5,11)=   0.00      A( 5,12)=   0.00
A( 5,13)=   0.00      A( 5,14)=   0.00      A( 5,15)=   0.00      A( 5,16)=   0.00      A( 5,17)=   0.00      A( 5,18)=   0.00
A( 5,19)=   0.00      A( 5,20)=   0.00      A( 5,21)=   0.00      A( 5,22)=   0.00      A( 5,23)=   0.00      A( 5,24)=   0.00
A( 6, 1)= -7.335E+06  A( 6, 2)=   0.00      A( 6, 3)=  2.667E+06  A( 6, 4)=  6.000E+07  A( 6, 5)= -1.346E+08  A( 6, 6)=  8.488E+08
A( 6, 7)= -7.335E+06  A( 6, 8)=   0.00      A( 6, 9)=  1.333E+07  A( 6,10)= -7.335E+06  A( 6,11)=   0.00      A( 6,12)=  1.867E+07
A( 6,13)= -7.335E+06  A( 6,14)=   0.00      A( 6,15)=  2.400E+07  A( 6,16)= -7.335E+06  A( 6,17)=   0.00      A( 6,18)=  2.934E+07
A( 6,19)= -7.335E+06  A( 6,20)=   0.00      A( 6,21)=  3.467E+07  A( 6,22)= -7.335E+06  A( 6,23)=   0.00      A( 6,24)=  4.000E+07
A( 7, 1)=  2.279E+06  A( 7, 2)=   0.00      A( 7, 3)= -2.445E+06  A( 7, 4)=  2.279E+06  A( 7, 5)=   0.00      A( 7, 6)= -7.335E+06
A( 7, 7)=  2.362E+10  A( 7, 8)= -4.691E+09  A( 7, 9)=  1.000E+08  A( 7,10)=  2.279E+06  A( 7,11)=   0.00      A( 7,12)= -1.711E+07
A( 7,13)=  2.279E+06  A( 7,14)=   0.00      A( 7,15)= -2.200E+07  A( 7,16)=  2.279E+06  A( 7,17)=   0.00      A( 7,18)= -2.689E+07
A( 7,19)=  2.279E+06  A( 7,20)=   0.00      A( 7,21)= -3.178E+07  A( 7,22)=  2.279E+06  A( 7,23)=   0.00      A( 7,24)= -3.667E+07
A( 8, 1)=   0.00      A( 8, 2)=   0.00      A( 8, 3)=   0.00      A( 8, 4)=   0.00      A( 8, 5)=   0.00      A( 8, 6)=   0.00
A( 8, 7)= -4.691E+09  A( 8, 8)=  1.041E+10  A( 8, 9)= -1.445E+08  A( 8,10)=   0.00      A( 8,11)=   0.00      A( 8,12)=   0.00
A( 8,13)=   0.00      A( 8,14)=   0.00      A( 8,15)=   0.00      A( 8,16)=   0.00      A( 8,17)=   0.00      A( 8,18)=   0.00
A( 8,19)=   0.00      A( 8,20)=   0.00      A( 8,21)=   0.00      A( 8,22)=   0.00      A( 8,23)=   0.00      A( 8,24)=   0.00
A( 9, 1)= -1.222E+07  A( 9, 2)=   0.00      A( 9, 3)=  4.445E+06  A( 9, 4)= -1.222E+07  A( 9, 5)=   0.00      A( 9, 6)=  1.333E+07
A( 9, 7)=  1.000E+08  A( 9, 8)= -1.445E+08  A( 9, 9)=  4.753E+09  A( 9,10)= -1.222E+07  A( 9,11)=   0.00      A( 9,12)=  3.111E+07
A( 9,13)= -1.222E+07  A( 9,14)=   0.00      A( 9,15)=  4.000E+07  A( 9,16)= -1.222E+07  A( 9,17)=   0.00      A( 9,18)=  4.889E+07
A( 9,19)= -1.222E+07  A( 9,20)=   0.00      A( 9,21)=  5.778E+07  A( 9,22)= -1.222E+07  A( 9,23)=   0.00      A( 9,24)=  6.667E+07
A(10, 1)=  2.279E+06  A(10, 2)=   0.00      A(10, 3)= -2.445E+06  A(10, 4)=  2.279E+06  A(10, 5)=   0.00      A(10, 6)= -7.335E+06
```

A(10, 7)= 2.279E+06	A(10, 8)= 0.00	A(10, 9)= -1.222E+07	A(10,10)= 4.550E+10	A(10,11)= -6.568E+09	A(10,12)= 1.400E+08
A(10,13)= 2.279E+06	A(10,14)= 0.00	A(10,15)= -2.200E+07	A(10,16)= 2.279E+06	A(10,17)= 0.00	A(10,18)= -2.689E+07
A(10,19)= 2.279E+06	A(10,20)= 0.00	A(10,21)= -3.178E+07	A(10,22)= 2.279E+06	A(10,23)= 0.00	A(10,24)= -3.667E+07
A(11, 1)= 0.00	A(11, 2)= 0.00	A(11, 3)= 0.00	A(11, 4)= 0.00	A(11, 5)= 0.00	A(11, 6)= 0.00
A(11, 7)= 0.00	A(11, 8)= 0.00	A(11, 9)= 0.00	A(11,10)= -6.568E+09	A(11,11)= 1.818E+10	A(11,12)= -1.594E+08
A(11,13)= 0.00	A(11,14)= 0.00	A(11,15)= 0.00	A(11,16)= 0.00	A(11,17)= 0.00	A(11,18)= 0.00
A(11,19)= 0.00	A(11,20)= 0.00	A(11,21)= 0.00	A(11,22)= 0.00	A(11,23)= 0.00	A(11,24)= 0.00
A(12, 1)= -1.711E+07	A(12, 2)= 0.00	A(12, 3)= 6.223E+06	A(12, 4)= -1.711E+07	A(12, 5)= 0.00	A(12, 6)= 1.867E+07
A(12, 7)= -1.711E+07	A(12, 8)= 0.00	A(12, 9)= 3.111E+07	A(12,10)= 1.400E+08	A(12,11)= -1.594E+08	A(12,12)= 1.656E+10
A(12,13)= -1.711E+07	A(12,14)= 0.00	A(12,15)= 5.601E+07	A(12,16)= -1.711E+07	A(12,17)= 0.00	A(12,18)= 6.845E+07
A(12,19)= -1.711E+07	A(12,20)= 0.00	A(12,21)= 8.090E+07	A(12,22)= -1.711E+07	A(12,23)= 0.00	A(12,24)= 9.334E+07
A(13, 1)= 2.279E+06	A(13, 2)= 0.00	A(13, 3)= -2.445E+06	A(13, 4)= 2.279E+06	A(13, 5)= 0.00	A(13, 6)= -7.335E+06
A(13, 7)= 2.279E+06	A(13, 8)= 0.00	A(13, 9)= -1.222E+07	A(13,10)= 2.279E+06	A(13,11)= 0.00	A(13,12)= -1.711E+07
A(13,13)= 7.468E+10	A(13,14)= -8.445E+09	A(13,15)= 1.800E+08	A(13,16)= 2.279E+06	A(13,17)= 0.00	A(13,18)= -2.689E+07
A(13,19)= 2.279E+06	A(13,20)= 0.00	A(13,21)= -3.178E+07	A(13,22)= 2.279E+06	A(13,23)= 0.00	A(13,24)= -3.667E+07
A(14, 1)= 0.00	A(14, 2)= 0.00	A(14, 3)= 0.00	A(14, 4)= 0.00	A(14, 5)= 0.00	A(14, 6)= 0.00
A(14, 7)= 0.00	A(14, 8)= 0.00	A(14, 9)= 0.00	A(14,10)= 0.00	A(14,11)= 0.00	A(14,12)= 0.00
A(14,13)= -8.445E+09	A(14,14)= 2.854E+10	A(14,15)= -1.794E+08	A(14,16)= 0.00	A(14,17)= 0.00	A(14,18)= 0.00
A(14,19)= 0.00	A(14,20)= 0.00	A(14,21)= 0.00	A(14,22)= 0.00	A(14,23)= 0.00	A(14,24)= 0.00
A(15, 1)= -2.200E+07	A(15, 2)= 0.00	A(15, 3)= 8.001E+06	A(15, 4)= -2.200E+07	A(15, 5)= 0.00	A(15, 6)= 2.400E+07
A(15, 7)= -2.200E+07	A(15, 8)= 0.00	A(15, 9)= 4.000E+07	A(15,10)= -2.200E+07	A(15,11)= 0.00	A(15,12)= 5.601E+07
A(15,13)= 1.800E+08	A(15,14)= -1.794E+08	A(15,15)= 4.341E+10	A(15,16)= -2.200E+07	A(15,17)= 0.00	A(15,18)= 8.801E+07
A(15,19)= -2.200E+07	A(15,20)= 0.00	A(15,21)= 1.040E+08	A(15,22)= -2.200E+07	A(15,23)= 0.00	A(15,24)= 1.200E+08
A(16, 1)= 2.279E+06	A(16, 2)= 0.00	A(16, 3)= -2.445E+06	A(16, 4)= 2.279E+06	A(16, 5)= 0.00	A(16, 6)= -7.335E+06
A(16, 7)= 2.279E+06	A(16, 8)= 0.00	A(16, 9)= -1.222E+07	A(16,10)= 2.279E+06	A(16,11)= 0.00	A(16,12)= -1.711E+07
A(16,13)= 2.279E+06	A(16,14)= 0.00	A(16,15)= -2.200E+07	A(16,16)= 1.111E+11	A(16,17)= -1.032E+10	A(16,18)= 2.200E+08
A(16,19)= 2.279E+06	A(16,20)= 0.00	A(16,21)= -3.178E+07	A(16,22)= 2.279E+06	A(16,23)= 0.00	A(16,24)= -3.667E+07
A(17, 1)= 0.00	A(17, 2)= 0.00	A(17, 3)= 0.00	A(17, 4)= 0.00	A(17, 5)= 0.00	A(17, 6)= 0.00
A(17, 7)= 0.00	A(17, 8)= 0.00	A(17, 9)= 0.00	A(17,10)= 0.00	A(17,11)= 0.00	A(17,12)= 0.00
A(17,13)= 0.00	A(17,14)= 0.00	A(17,15)= 0.00	A(17,16)= -1.032E+10	A(17,17)= 4.149E+10	A(17,18)= -2.043E+08
A(17,19)= 0.00	A(17,20)= 0.00	A(17,21)= 0.00	A(17,22)= 0.00	A(17,23)= 0.00	A(17,24)= 0.00
A(18, 1)= -2.689E+07	A(18, 2)= 0.00	A(18, 3)= 9.779E+06	A(18, 4)= -2.689E+07	A(18, 5)= 0.00	A(18, 6)= 2.934E+07
A(18, 7)= -2.689E+07	A(18, 8)= 0.00	A(18, 9)= 4.889E+07	A(18,10)= -2.689E+07	A(18,11)= 0.00	A(18,12)= 6.845E+07
A(18,13)= -2.689E+07	A(18,14)= 0.00	A(18,15)= 8.801E+07	A(18,16)= 2.200E+08	A(18,17)= -2.043E+08	A(18,18)= 9.481E+10
A(18,19)= -2.689E+07	A(18,20)= 0.00	A(18,21)= 1.271E+08	A(18,22)= -2.689E+07	A(18,23)= 0.00	A(18,24)= 1.467E+08
A(19, 1)= 2.279E+06	A(19, 2)= 0.00	A(19, 3)= -2.445E+06	A(19, 4)= 2.279E+06	A(19, 5)= 0.00	A(19, 6)= -7.335E+06
A(19, 7)= 2.279E+06	A(19, 8)= 0.00	A(19, 9)= -1.222E+07	A(19,10)= 2.279E+06	A(19,11)= 0.00	A(19,12)= -1.711E+07
A(19,13)= 2.279E+06	A(19,14)= 0.00	A(19,15)= -2.200E+07	A(19,16)= 2.279E+06	A(19,17)= 0.00	A(19,18)= -2.689E+07
A(19,19)= 1.549E+11	A(19,20)= -1.220E+10	A(19,21)= 2.600E+08	A(19,22)= 2.279E+06	A(19,23)= 0.00	A(19,24)= -3.667E+07
A(20, 1)= 0.00	A(20, 2)= 0.00	A(20, 3)= 0.00	A(20, 4)= 0.00	A(20, 5)= 0.00	A(20, 6)= 0.00
A(20, 7)= 0.00	A(20, 8)= 0.00	A(20, 9)= 0.00	A(20,10)= 0.00	A(20,11)= 0.00	A(20,12)= 0.00
A(20,13)= 0.00	A(20,14)= 0.00	A(20,15)= 0.00	A(20,16)= 0.00	A(20,17)= 0.00	A(20,18)= 0.00
A(20,19)= -1.220E+10	A(20,20)= 5.702E+10	A(20,21)= -2.341E+08	A(20,22)= 0.00	A(20,23)= 0.00	A(20,24)= 0.00
A(21, 1)= -3.178E+07	A(21, 2)= 0.00	A(21, 3)= 1.156E+07	A(21, 4)= -3.178E+07	A(21, 5)= 0.00	A(21, 6)= 3.467E+07
A(21, 7)= -3.178E+07	A(21, 8)= 0.00	A(21, 9)= 5.778E+07	A(21,10)= -3.178E+07	A(21,11)= 0.00	A(21,12)= 8.090E+07
A(21,13)= -3.178E+07	A(21,14)= 0.00	A(21,15)= 1.040E+08	A(21,16)= -3.178E+07	A(21,17)= 0.00	A(21,18)= 1.271E+08
A(21,19)= 2.600E+08	A(21,20)= -2.341E+08	A(21,21)= 1.827E+11	A(21,22)= -3.178E+07	A(21,23)= 0.00	A(21,24)= 1.733E+08
A(22, 1)= 2.279E+06	A(22, 2)= 0.00	A(22, 3)= -2.445E+06	A(22, 4)= 2.279E+06	A(22, 5)= 0.00	A(22, 6)= -7.335E+06
A(22, 7)= 2.279E+06	A(22, 8)= 0.00	A(22, 9)= -1.222E+07	A(22,10)= 2.279E+06	A(22,11)= 0.00	A(22,12)= -1.711E+07
A(22,13)= 2.279E+06	A(22,14)= 0.00	A(22,15)= -2.200E+07	A(22,16)= 2.279E+06	A(22,17)= 0.00	A(22,18)= -2.689E+07
A(22,19)= 2.279E+06	A(22,20)= 0.00	A(22,21)= -3.178E+07	A(22,22)= 2.060E+11	A(22,23)= -1.407E+10	A(22,24)= 3.000E+08
A(23, 1)= 0.00	A(23, 2)= 0.00	A(23, 3)= 0.00	A(23, 4)= 0.00	A(23, 5)= 0.00	A(23, 6)= 0.00
A(23, 7)= 0.00	A(23, 8)= 0.00	A(23, 9)= 0.00	A(23,10)= 0.00	A(23,11)= 0.00	A(23,12)= 0.00
A(23,13)= 0.00	A(23,14)= 0.00	A(23,15)= 0.00	A(23,16)= 0.00	A(23,17)= 0.00	A(23,18)= 0.00

A(23,19)= 0.00	A(23,20)= 0.00	A(23,21)= 0.00	A(23,22)= -1.407E+10	A(23,23)= 7.515E+10	A(23,24)= -2.690E+08
A(24, 1)= -3.667E+07	A(24, 2)= 0.00	A(24, 3)= 1.333E+07	A(24, 4)= -3.667E+07	A(24, 5)= 0.00	A(24, 6)= 4.000E+07
A(24, 7)= -3.667E+07	A(24, 8)= 0.00	A(24, 9)= 6.667E+07	A(24,10)= -3.667E+07	A(24,11)= 0.00	A(24,12)= 9.334E+07
A(24,13)= -3.667E+07	A(24,14)= 0.00	A(24,15)= 1.200E+08	A(24,16)= -3.667E+07	A(24,17)= 0.00	A(24,18)= 1.467E+08
A(24,19)= -3.667E+07	A(24,20)= 0.00	A(24,21)= 1.733E+08	A(24,22)= 3.000E+08	A(24,23)= -2.690E+08	A(24,24)= 3.213E+11
B(1, 1)= 2.902E+03	B(1, 2)= 877.	B(1, 3)= -97.5	B(1, 4)= 0.00	B(1, 5)= 0.00	B(1, 6)= 0.00
B(1, 7)= 0.00	B(1, 8)= 0.00	B(1, 9)= 0.00	B(1,10)= 0.00	B(1,11)= 0.00	B(1,12)= 0.00
B(1,13)= 0.00	B(1,14)= 0.00	B(1,15)= 0.00	B(1,16)= 0.00	B(1,17)= 0.00	B(1,18)= 0.00
B(1,19)= 0.00	B(1,20)= 0.00	B(1,21)= 0.00	B(1,22)= 0.00	B(1,23)= 0.00	B(1,24)= 0.00
B(2, 1)= 877.	B(2, 2)= 564.	B(2, 3)= 13.1	B(2, 4)= 0.00	B(2, 5)= 0.00	B(2, 6)= 0.00
B(2, 7)= 0.00	B(2, 8)= 0.00	B(2, 9)= 0.00	B(2,10)= 0.00	B(2,11)= 0.00	B(2,12)= 0.00
B(2,13)= 0.00	B(2,14)= 0.00	B(2,15)= 0.00	B(2,16)= 0.00	B(2,17)= 0.00	B(2,18)= 0.00
B(2,19)= 0.00	B(2,20)= 0.00	B(2,21)= 0.00	B(2,22)= 0.00	B(2,23)= 0.00	B(2,24)= 0.00
B(3, 1)= -97.5	B(3, 2)= 13.1	B(3, 3)= 3.119E+03	B(3, 4)= 0.00	B(3, 5)= 0.00	B(3, 6)= 0.00
B(3, 7)= 0.00	B(3, 8)= 0.00	B(3, 9)= 0.00	B(3,10)= 0.00	B(3,11)= 0.00	B(3,12)= 0.00
B(3,13)= 0.00	B(3,14)= 0.00	B(3,15)= 0.00	B(3,16)= 0.00	B(3,17)= 0.00	B(3,18)= 0.00
B(3,19)= 0.00	B(3,20)= 0.00	B(3,21)= 0.00	B(3,22)= 0.00	B(3,23)= 0.00	B(3,24)= 0.00
B(4, 1)= 0.00	B(4, 2)= 0.00	B(4, 3)= 0.00	B(4, 4)= 2.902E+03	B(4, 5)= 2.631E+03	B(4, 6)= -292.
B(4, 7)= 0.00	B(4, 8)= 0.00	B(4, 9)= 0.00	B(4,10)= 0.00	B(4,11)= 0.00	B(4,12)= 0.00
B(4,13)= 0.00	B(4,14)= 0.00	B(4,15)= 0.00	B(4,16)= 0.00	B(4,17)= 0.00	B(4,18)= 0.00
B(4,19)= 0.00	B(4,20)= 0.00	B(4,21)= 0.00	B(4,22)= 0.00	B(4,23)= 0.00	B(4,24)= 0.00
B(5, 1)= 0.00	B(5, 2)= 0.00	B(5, 3)= 0.00	B(5, 4)= 2.631E+03	B(5, 5)= 5.074E+03	B(5, 6)= 13.1
B(5, 7)= 0.00	B(5, 8)= 0.00	B(5, 9)= 0.00	B(5,10)= 0.00	B(5,11)= 0.00	B(5,12)= 0.00
B(5,13)= 0.00	B(5,14)= 0.00	B(5,15)= 0.00	B(5,16)= 0.00	B(5,17)= 0.00	B(5,18)= 0.00
B(5,19)= 0.00	B(5,20)= 0.00	B(5,21)= 0.00	B(5,22)= 0.00	B(5,23)= 0.00	B(5,24)= 0.00
B(6, 1)= 0.00	B(6, 2)= 0.00	B(6, 3)= 0.00	B(6, 4)= -292.	B(6, 5)= 13.1	B(6, 6)= 7.629E+03
B(6, 7)= 0.00	B(6, 8)= 0.00	B(6, 9)= 0.00	B(6,10)= 0.00	B(6,11)= 0.00	B(6,12)= 0.00
B(6,13)= 0.00	B(6,14)= 0.00	B(6,15)= 0.00	B(6,16)= 0.00	B(6,17)= 0.00	B(6,18)= 0.00
B(6,19)= 0.00	B(6,20)= 0.00	B(6,21)= 0.00	B(6,22)= 0.00	B(6,23)= 0.00	B(6,24)= 0.00
B(7, 1)= 0.00	B(7, 2)= 0.00	B(7, 3)= 0.00	B(7, 4)= 0.00	B(7, 5)= 0.00	B(7, 6)= 0.00
B(7, 7)= 2.902E+03	B(7, 8)= 4.386E+03	B(7, 9)= -487.	B(7,10)= 0.00	B(7,11)= 0.00	B(7,12)= 0.00
B(7,13)= 0.00	B(7,14)= 0.00	B(7,15)= 0.00	B(7,16)= 0.00	B(7,17)= 0.00	B(7,18)= 0.00
B(7,19)= 0.00	B(7,20)= 0.00	B(7,21)= 0.00	B(7,22)= 0.00	B(7,23)= 0.00	B(7,24)= 0.00
B(8, 1)= 0.00	B(8, 2)= 0.00	B(8, 3)= 0.00	B(8, 4)= 0.00	B(8, 5)= 0.00	B(8, 6)= 0.00
B(8, 7)= 4.386E+03	B(8, 8)= 1.409E+04	B(8, 9)= 13.1	B(8,10)= 0.00	B(8,11)= 0.00	B(8,12)= 0.00
B(8,13)= 0.00	B(8,14)= 0.00	B(8,15)= 0.00	B(8,16)= 0.00	B(8,17)= 0.00	B(8,18)= 0.00
B(8,19)= 0.00	B(8,20)= 0.00	B(8,21)= 0.00	B(8,22)= 0.00	B(8,23)= 0.00	B(8,24)= 0.00
B(9, 1)= 0.00	B(9, 2)= 0.00	B(9, 3)= 0.00	B(9, 4)= 0.00	B(9, 5)= 0.00	B(9, 6)= 0.00
B(9, 7)= -487.	B(9, 8)= 13.1	B(9, 9)= 1.665E+04	B(9,10)= 0.00	B(9,11)= 0.00	B(9,12)= 0.00
B(9,13)= 0.00	B(9,14)= 0.00	B(9,15)= 0.00	B(9,16)= 0.00	B(9,17)= 0.00	B(9,18)= 0.00
B(9,19)= 0.00	B(9,20)= 0.00	B(9,21)= 0.00	B(9,22)= 0.00	B(9,23)= 0.00	B(9,24)= 0.00
B(10, 1)= 0.00	B(10, 2)= 0.00	B(10, 3)= 0.00	B(10, 4)= 0.00	B(10, 5)= 0.00	B(10, 6)= 0.00
B(10, 7)= 0.00	B(10, 8)= 0.00	B(10, 9)= 0.00	B(10,10)= 2.902E+03	B(10,11)= 6.140E+03	B(10,12)= -682.
B(10,13)= 0.00	B(10,14)= 0.00	B(10,15)= 0.00	B(10,16)= 0.00	B(10,17)= 0.00	B(10,18)= 0.00
B(10,19)= 0.00	B(10,20)= 0.00	B(10,21)= 0.00	B(10,22)= 0.00	B(10,23)= 0.00	B(10,24)= 0.00
B(11, 1)= 0.00	B(11, 2)= 0.00	B(11, 3)= 0.00	B(11, 4)= 0.00	B(11, 5)= 0.00	B(11, 6)= 0.00
B(11, 7)= 0.00	B(11, 8)= 0.00	B(11, 9)= 0.00	B(11,10)= 6.140E+03	B(11,11)= 2.762E+04	B(11,12)= 13.1
B(11,13)= 0.00	B(11,14)= 0.00	B(11,15)= 0.00	B(11,16)= 0.00	B(11,17)= 0.00	B(11,18)= 0.00
B(11,19)= 0.00	B(11,20)= 0.00	B(11,21)= 0.00	B(11,22)= 0.00	B(11,23)= 0.00	B(11,24)= 0.00
B(12, 1)= 0.00	B(12, 2)= 0.00	B(12, 3)= 0.00	B(12, 4)= 0.00	B(12, 5)= 0.00	B(12, 6)= 0.00
B(12, 7)= 0.00	B(12, 8)= 0.00	B(12, 9)= 0.00	B(12,10)= -682.	B(12,11)= 13.1	B(12,12)= 3.018E+04
B(12,13)= 0.00	B(12,14)= 0.00	B(12,15)= 0.00	B(12,16)= 0.00	B(12,17)= 0.00	B(12,18)= 0.00
B(12,19)= 0.00	B(12,20)= 0.00	B(12,21)= 0.00	B(12,22)= 0.00	B(12,23)= 0.00	B(12,24)= 0.00
B(13, 1)= 0.00	B(13, 2)= 0.00	B(13, 3)= 0.00	B(13, 4)= 0.00	B(13, 5)= 0.00	B(13, 6)= 0.00

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B(13, 7)=   0.00      B(13, 8)=   0.00      B(13, 9)=   0.00      B(13,10)=   0.00      B(13,11)=   0.00      B(13,12)=   0.00
B(13,13)=  2.902E+03  B(13,14)=  7.894E+03  B(13,15)=  -877.      B(13,16)=   0.00      B(13,17)=   0.00      B(13,18)=   0.00
B(13,19)=   0.00      B(13,20)=   0.00      B(13,21)=   0.00      B(13,22)=   0.00      B(13,23)=   0.00      B(13,24)=   0.00
B(14, 1)=   0.00      B(14, 2)=   0.00      B(14, 3)=   0.00      B(14, 4)=   0.00      B(14, 5)=   0.00      B(14, 6)=   0.00
B(14, 7)=   0.00      B(14, 8)=   0.00      B(14, 9)=   0.00      B(14,10)=   0.00      B(14,11)=   0.00      B(14,12)=   0.00
B(14,13)=  7.894E+03  B(14,14)=  4.566E+04  B(14,15)=   13.1      B(14,16)=   0.00      B(14,17)=   0.00      B(14,18)=   0.00
B(14,19)=   0.00      B(14,20)=   0.00      B(14,21)=   0.00      B(14,22)=   0.00      B(14,23)=   0.00      B(14,24)=   0.00
B(15, 1)=   0.00      B(15, 2)=   0.00      B(15, 3)=   0.00      B(15, 4)=   0.00      B(15, 5)=   0.00      B(15, 6)=   0.00
B(15, 7)=   0.00      B(15, 8)=   0.00      B(15, 9)=   0.00      B(15,10)=   0.00      B(15,11)=   0.00      B(15,12)=   0.00
B(15,13)=  -877.      B(15,14)=   13.1      B(15,15)=  4.822E+04  B(15,16)=   0.00      B(15,17)=   0.00      B(15,18)=   0.00
B(15,19)=   0.00      B(15,20)=   0.00      B(15,21)=   0.00      B(15,22)=   0.00      B(15,23)=   0.00      B(15,24)=   0.00
B(16, 1)=   0.00      B(16, 2)=   0.00      B(16, 3)=   0.00      B(16, 4)=   0.00      B(16, 5)=   0.00      B(16, 6)=   0.00
B(16, 7)=   0.00      B(16, 8)=   0.00      B(16, 9)=   0.00      B(16,10)=   0.00      B(16,11)=   0.00      B(16,12)=   0.00
B(16,13)=   0.00      B(16,14)=   0.00      B(16,15)=   0.00      B(16,16)=  2.902E+03  B(16,17)=  9.649E+03  B(16,18)= -1.072E+03
B(16,19)=   0.00      B(16,20)=   0.00      B(16,21)=   0.00      B(16,22)=   0.00      B(16,23)=   0.00      B(16,24)=   0.00
B(17, 1)=   0.00      B(17, 2)=   0.00      B(17, 3)=   0.00      B(17, 4)=   0.00      B(17, 5)=   0.00      B(17, 6)=   0.00
B(17, 7)=   0.00      B(17, 8)=   0.00      B(17, 9)=   0.00      B(17,10)=   0.00      B(17,11)=   0.00      B(17,12)=   0.00
B(17,13)=   0.00      B(17,14)=   0.00      B(17,15)=   0.00      B(17,16)=  9.649E+03  B(17,17)=  6.821E+04  B(17,18)=   13.1
B(17,19)=   0.00      B(17,20)=   0.00      B(17,21)=   0.00      B(17,22)=   0.00      B(17,23)=   0.00      B(17,24)=   0.00
B(18, 1)=   0.00      B(18, 2)=   0.00      B(18, 3)=   0.00      B(18, 4)=   0.00      B(18, 5)=   0.00      B(18, 6)=   0.00
B(18, 7)=   0.00      B(18, 8)=   0.00      B(18, 9)=   0.00      B(18,10)=   0.00      B(18,11)=   0.00      B(18,12)=   0.00
B(18,13)=   0.00      B(18,14)=   0.00      B(18,15)=   0.00      B(18,16)= -1.072E+03  B(18,17)=   13.1      B(18,18)=  7.077E+04
B(18,19)=   0.00      B(18,20)=   0.00      B(18,21)=   0.00      B(18,22)=   0.00      B(18,23)=   0.00      B(18,24)=   0.00
B(19, 1)=   0.00      B(19, 2)=   0.00      B(19, 3)=   0.00      B(19, 4)=   0.00      B(19, 5)=   0.00      B(19, 6)=   0.00
B(19, 7)=   0.00      B(19, 8)=   0.00      B(19, 9)=   0.00      B(19,10)=   0.00      B(19,11)=   0.00      B(19,12)=   0.00
B(19,13)=   0.00      B(19,14)=   0.00      B(19,15)=   0.00      B(19,16)=   0.00      B(19,17)=   0.00      B(19,18)=   0.00
B(19,19)=  2.902E+03  B(19,20)=  1.140E+04  B(19,21)= -1.267E+03  B(19,22)=   0.00      B(19,23)=   0.00      B(19,24)=   0.00
B(20, 1)=   0.00      B(20, 2)=   0.00      B(20, 3)=   0.00      B(20, 4)=   0.00      B(20, 5)=   0.00      B(20, 6)=   0.00
B(20, 7)=   0.00      B(20, 8)=   0.00      B(20, 9)=   0.00      B(20,10)=   0.00      B(20,11)=   0.00      B(20,12)=   0.00
B(20,13)=   0.00      B(20,14)=   0.00      B(20,15)=   0.00      B(20,16)=   0.00      B(20,17)=   0.00      B(20,18)=   0.00
B(20,19)=  1.140E+04  B(20,20)=  9.527E+04  B(20,21)=   13.1      B(20,22)=   0.00      B(20,23)=   0.00      B(20,24)=   0.00
B(21, 1)=   0.00      B(21, 2)=   0.00      B(21, 3)=   0.00      B(21, 4)=   0.00      B(21, 5)=   0.00      B(21, 6)=   0.00
B(21, 7)=   0.00      B(21, 8)=   0.00      B(21, 9)=   0.00      B(21,10)=   0.00      B(21,11)=   0.00      B(21,12)=   0.00
B(21,13)=   0.00      B(21,14)=   0.00      B(21,15)=   0.00      B(21,16)=   0.00      B(21,17)=   0.00      B(21,18)=   0.00
B(21,19)= -1.267E+03  B(21,20)=   13.1      B(21,21)=  9.783E+04  B(21,22)=   0.00      B(21,23)=   0.00      B(21,24)=   0.00
B(22, 1)=   0.00      B(22, 2)=   0.00      B(22, 3)=   0.00      B(22, 4)=   0.00      B(22, 5)=   0.00      B(22, 6)=   0.00
B(22, 7)=   0.00      B(22, 8)=   0.00      B(22, 9)=   0.00      B(22,10)=   0.00      B(22,11)=   0.00      B(22,12)=   0.00
B(22,13)=   0.00      B(22,14)=   0.00      B(22,15)=   0.00      B(22,16)=   0.00      B(22,17)=   0.00      B(22,18)=   0.00
B(22,19)=   0.00      B(22,20)=   0.00      B(22,21)=   0.00      B(22,22)=  2.902E+03  B(22,23)=  1.316E+04  B(22,24)= -1.462E+03
B(23, 1)=   0.00      B(23, 2)=   0.00      B(23, 3)=   0.00      B(23, 4)=   0.00      B(23, 5)=   0.00      B(23, 6)=   0.00
B(23, 7)=   0.00      B(23, 8)=   0.00      B(23, 9)=   0.00      B(23,10)=   0.00      B(23,11)=   0.00      B(23,12)=   0.00
B(23,13)=   0.00      B(23,14)=   0.00      B(23,15)=   0.00      B(23,16)=   0.00      B(23,17)=   0.00      B(23,18)=   0.00
B(23,19)=   0.00      B(23,20)=   0.00      B(23,21)=   0.00      B(23,22)=  1.316E+04  B(23,23)=  1.268E+05  B(23,24)=   13.1
B(24, 1)=   0.00      B(24, 2)=   0.00      B(24, 3)=   0.00      B(24, 4)=   0.00      B(24, 5)=   0.00      B(24, 6)=   0.00
B(24, 7)=   0.00      B(24, 8)=   0.00      B(24, 9)=   0.00      B(24,10)=   0.00      B(24,11)=   0.00      B(24,12)=   0.00
B(24,13)=   0.00      B(24,14)=   0.00      B(24,15)=   0.00      B(24,16)=   0.00      B(24,17)=   0.00      B(24,18)=   0.00
B(24,19)=   0.00      B(24,20)=   0.00      B(24,21)=   0.00      B(24,22)= -1.462E+03  B(24,23)=   13.1      B(24,24)=  1.294E+05
 THE PERFORMANCE INDEX IS   2.48
 Ring-Shell Stiffness Factor =  7.049E-04

 ELASTIC INTER-BAY INSTABILITY PRESSURE =  1.592E+04   MODE =  8
 ***  INELASTIC BUCKLING PRESSURE =  1.221E+03 PSI.
 THE STRESS LEVEL IN THE SHELL =   108.0931 PERCENT YIELD.
 (The pseudo-plastic curves did not intersect.)
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ELASTIC MODE SHAPE:
U(1) = 8.255E-03 V(1) = 7.888E-02 W(1) = 0.997
U(2) = 1.952E-05 V(2) = -2.578E-06 W(2) = -5.554E-04
U(3) = 6.140E-06 V(3) = 6.655E-07 W(3) = -1.159E-04
U(4) = 2.978E-06 V(4) = 3.686E-07 W(4) = -4.183E-05
U(5) = 1.760E-06 V(5) = 1.981E-07 W(5) = -1.961E-05
U(6) = 1.164E-06 V(6) = 1.153E-07 W(6) = -1.072E-05
U(7) = 8.274E-07 V(7) = 7.227E-08 W(7) = -6.487E-06
U(8) = 6.186E-07 V(8) = 4.802E-08 W(8) = -4.220E-06
(THE FOLLOWING IS REFERENCE INFORMATION ONLY:
ELASTIC BUCKLING PRESSURE = 1.514E+04 ASSUMING SIMPLE SUPPORT BOUNDARY
CONDITIONS AT THE FRAME EDGE. MODE = 9 THIS DAPS3-TYPE CALCULATION
USES LS = 2.650 = "UNSUPPORTED BAY WIDTH".)

0 SHELL WALL STIFFNESS COEFFICIENTS
A11 = 8.189E+06 A22 = 8.189E+06 A12 = 2.375E+06 A66 = 2.907E+06
D11 = 4.265E+04 D22 = 4.265E+04 D12 = 1.237E+04 D66 = 3.028E+04

PARAMETERS FOR STRESSES AND DEFLECTIONS
THETA = 2.17 GAMMA = 4.420E-02 QSTAR = -308. DELTA = 2.080E+07 BETA = 0.260
F1 = 0.893 F2 = 0.800 F3 = -0.687 F4 = 0.336 F5 = 0.372 F6 = 0.816

AUGMENTED MATRIX FOR ELASTIC INTERBAY BUCKLING WITH SIMPLE SUPPORT
A1 = 1.413E+09 A2 = -6.183E+08 A3 = 2.780E+07 A4 = 1.036E+10 A5 = 4.935E+09
A6 = -5.020E+08 A7 = -1.039E+10 A8 = -4.852E+09 A9 = 4.710E+08 A10 = 1.654E+03

POINT	STATION	W	SLOPE	CURVATURE	SIGX(IN)	SIGX(OUT)	SIGY(IN)	SIGY(OUT)	SVON(IN)	SVON(OUT)
1	0.00	-1.009E-02	0.00	-5.511E-04	-1.800E+04	-2.251E+04	-3.588E+04	-3.719E+04	3.107E+04	3.244E+04
2	3.786E-02	-1.009E-02	2.085E-05	-5.498E-04	-1.800E+04	-2.250E+04	-3.588E+04	-3.718E+04	3.107E+04	3.244E+04
3	7.571E-02	-1.009E-02	4.160E-05	-5.459E-04	-1.802E+04	-2.249E+04	-3.588E+04	-3.717E+04	3.107E+04	3.243E+04
4	0.114	-1.009E-02	6.215E-05	-5.393E-04	-1.804E+04	-2.246E+04	-3.588E+04	-3.716E+04	3.107E+04	3.242E+04
5	0.151	-1.009E-02	8.240E-05	-5.302E-04	-1.808E+04	-2.242E+04	-3.588E+04	-3.714E+04	3.108E+04	3.240E+04
6	0.189	-1.008E-02	1.023E-04	-5.183E-04	-1.813E+04	-2.238E+04	-3.589E+04	-3.712E+04	3.108E+04	3.237E+04
7	0.227	-1.008E-02	1.216E-04	-5.039E-04	-1.819E+04	-2.232E+04	-3.589E+04	-3.709E+04	3.108E+04	3.234E+04
8	0.265	-1.007E-02	1.404E-04	-4.868E-04	-1.826E+04	-2.225E+04	-3.590E+04	-3.705E+04	3.109E+04	3.230E+04
9	0.303	-1.007E-02	1.584E-04	-4.670E-04	-1.834E+04	-2.217E+04	-3.590E+04	-3.701E+04	3.109E+04	3.226E+04
10	0.341	-1.006E-02	1.757E-04	-4.446E-04	-1.843E+04	-2.207E+04	-3.591E+04	-3.696E+04	3.110E+04	3.221E+04
11	0.379	-1.005E-02	1.921E-04	-4.195E-04	-1.854E+04	-2.197E+04	-3.592E+04	-3.691E+04	3.111E+04	3.216E+04
12	0.416	-1.005E-02	2.074E-04	-3.917E-04	-1.865E+04	-2.186E+04	-3.593E+04	-3.686E+04	3.112E+04	3.210E+04
13	0.454	-1.004E-02	2.217E-04	-3.612E-04	-1.877E+04	-2.173E+04	-3.594E+04	-3.680E+04	3.113E+04	3.204E+04
14	0.492	-1.003E-02	2.347E-04	-3.279E-04	-1.891E+04	-2.160E+04	-3.595E+04	-3.673E+04	3.115E+04	3.197E+04
15	0.530	-1.002E-02	2.465E-04	-2.920E-04	-1.906E+04	-2.145E+04	-3.597E+04	-3.666E+04	3.117E+04	3.190E+04
16	0.568	-1.001E-02	2.568E-04	-2.532E-04	-1.922E+04	-2.129E+04	-3.598E+04	-3.659E+04	3.119E+04	3.183E+04
17	0.606	-1.000E-02	2.656E-04	-2.117E-04	-1.939E+04	-2.112E+04	-3.600E+04	-3.651E+04	3.121E+04	3.175E+04
18	0.644	-9.990E-03	2.728E-04	-1.674E-04	-1.957E+04	-2.094E+04	-3.603E+04	-3.642E+04	3.124E+04	3.166E+04
19	0.681	-9.980E-03	2.783E-04	-1.202E-04	-1.976E+04	-2.075E+04	-3.605E+04	-3.634E+04	3.127E+04	3.157E+04
20	0.719	-9.969E-03	2.819E-04	-7.028E-05	-1.997E+04	-2.054E+04	-3.608E+04	-3.624E+04	3.130E+04	3.148E+04
21	0.757	-9.959E-03	2.835E-04	-1.746E-05	-2.018E+04	-2.032E+04	-3.611E+04	-3.615E+04	3.134E+04	3.139E+04
22	0.795	-9.948E-03	2.832E-04	3.823E-05	-2.041E+04	-2.010E+04	-3.614E+04	-3.605E+04	3.139E+04	3.129E+04
23	0.833	-9.937E-03	2.806E-04	9.682E-05	-2.065E+04	-1.986E+04	-3.618E+04	-3.595E+04	3.144E+04	3.119E+04
24	0.871	-9.927E-03	2.758E-04	1.583E-04	-2.090E+04	-1.961E+04	-3.622E+04	-3.584E+04	3.149E+04	3.109E+04
25	0.909	-9.916E-03	2.686E-04	2.227E-04	-2.117E+04	-1.934E+04	-3.626E+04	-3.573E+04	3.155E+04	3.098E+04
26	0.946	-9.906E-03	2.589E-04	2.901E-04	-2.144E+04	-1.907E+04	-3.631E+04	-3.562E+04	3.162E+04	3.088E+04
27	0.984	-9.897E-03	2.466E-04	3.604E-04	-2.173E+04	-1.878E+04	-3.637E+04	-3.551E+04	3.169E+04	3.077E+04

28	1.02	-9.888E-03	2.316E-04	4.337E-04	-2.203E+04	-1.848E+04	-3.643E+04	-3.540E+04	3.178E+04	3.066E+04
29	1.06	-9.879E-03	2.137E-04	5.099E-04	-2.234E+04	-1.817E+04	-3.649E+04	-3.528E+04	3.187E+04	3.056E+04
30	1.10	-9.872E-03	1.929E-04	5.892E-04	-2.267E+04	-1.784E+04	-3.656E+04	-3.516E+04	3.197E+04	3.045E+04
31	1.14	-9.865E-03	1.691E-04	6.714E-04	-2.300E+04	-1.750E+04	-3.664E+04	-3.505E+04	3.207E+04	3.035E+04
32	1.17	-9.859E-03	1.421E-04	7.566E-04	-2.335E+04	-1.716E+04	-3.672E+04	-3.493E+04	3.219E+04	3.025E+04
33	1.21	-9.854E-03	1.117E-04	8.449E-04	-2.371E+04	-1.679E+04	-3.681E+04	-3.481E+04	3.232E+04	3.015E+04
34	1.25	-9.850E-03	7.805E-05	9.361E-04	-2.409E+04	-1.642E+04	-3.691E+04	-3.469E+04	3.246E+04	3.005E+04
35	1.29	-9.848E-03	4.083E-05	1.030E-03	-2.447E+04	-1.603E+04	-3.702E+04	-3.457E+04	3.261E+04	2.996E+04
36	1.32	-9.847E-03	-8.348E-12	1.127E-03	-2.487E+04	-1.564E+04	-3.713E+04	-3.445E+04	3.277E+04	2.988E+04

THIS CASE TOOK 0.172 CP SECONDS.

References

1. Renzi, John R., “Axisymmetric Stresses and Deflections, Interbay Buckling, and General Instability of Orthotropic, Hybrid, Ring-Stiffened Cylindrical Shells under External Hydrostatic Pressure,” NSWC TR 80-269, 19 Apr 1981.
2. Renzi, John R., “Optimization of Orthotropic, Non-Linear, Ring-Stiffened Cylindrical Shells under External Hydrostatic Pressure As Applied to MMC Materials,” Naval Surface Weapons Center, NSWC TR 79-305, 30 Sep 1979.
3. Bushnell, David, “Computerized Analysis of Shells, Volume 2: Governing Equations,” Lockheed Palo Alto Research Laboratory, LMSC-D681421, Dec 1979.
4. Pulos, J. G., and Salerno, V. L., “Axisymmetric Elastic Deformations and Stresses in a Ring-Stiffened, Perfectly Circular Cylindrical Shell under External Hydrostatic Pressure,” David W. Taylor Model Basin Report 1497, Sep 1961.
5. Lunchick, M. E., “Yield Failure of Stiffened Cylinders under Hydrostatic Pressure,” Proceedings of the Third U. S. National Congress of Applied Mechanics, Jun 1958.
6. Lunchick, M. E., “Yield Failure of Stiffened Cylinders under Hydrostatic Pressure,” David Taylor Model Basin, DTMB Report 1291, Jan 1959.
7. Reynolds, Thomas E., “Inelastic Lobar Buckling of Cylindrical Shells under External Hydrostatic Pressure,” David W. Taylor Model Basin, Report 1392, Aug 1960.
8. Boichot, Lance and Reynolds, Thomas E., “Inelastic Buckling Tests of Ring-Stiffened Cylinders under Hydrostatic Pressure,” David Taylor Model Basin, DTMB Report 1992, May 1965.
9. Bushnell, David, “Buckling of Elastic-Plastic Shells of Revolution with Discrete Elastic-Plastic Ring Stiffeners,” *International Journal of Solids and Structures*, 1976, Vol. 12, pp. 51-66. Pergamon Press.

Appendix A

The following is the DAPS4 input deck for computing the general instability pressure for the Boichot and Reynolds tests of reference A-1. For this set of calculations, the average ring spacing, L_f, is input along with the actual shell length to best represent the overall shell stiffness (including the correct number of rings) for calculation of the general instability pressure. Since the input value for L_f is not the actual, the computed interbay buckling and axisymmetric collapse pressures and the stresses and deflections are ignored for this set of calculations.

Following this listing is the DAPS4 input deck for computing the interbay buckling pressure, the axisymmetric collapse pressure, and the stresses and deflections.

```
       1       1       0       0     1    1    0    8    18
       1 25-88     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.      0.3     0.35   0.
9450.000     0.    0.    0.   0
80600.00       0.        0.        0.
4.473   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.6390      0.   0.   0.   0.   0.
0.00   0.       0.127    0.386      0.   0.
80600.00       0       10       0       0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1       1       0       0     1    1    0    8    18
       2 25-86     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.      0.3     0.35   0.
9550.000     0.    0.    0.   0
80600.00       0.        0.        0.
4.349   2.1664   0.0832      0.101   0.   0.   0.
-1.0   0.6213      0.   0.   0.   0.   0.
0.00   0.       0.107    0.318      0.   0.
80600.00       0       10       0       0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1       1       0       0     1    1    0    8    18
       3 25-84     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.      0.3     0.35   0.
8850.000     0.    0.    0.   0
80700.00       0.        0.        0.
4.216   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.6023      0.   0.   0.   0.   0.
0.00   0.       0.086    0.255      0.   0.
80700.00       0       10       0       0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1       1       0       0     1    1    0    8    18
       4 25-82     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.      0.3     0.35   0.
7275.000     0.    0.    0.   0
80700.00       0.        0.        0.
4.047   2.1652   0.0826      0.101   0.   0.   0.
-1.0   0.5781      0.   0.   0.   0.   0.
0.00   0.       0.059    0.179      0.   0.
80700.00       0       10       0       0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1       1       0       0     1    1    0    8    18
       5 20-88     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.      0.3     0.35   0.
10600.00     0.    0.    0.   0
80700.00       0.        0.        0.
3.647   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.5210      0.   0.   0.   0.   0.
```

```
0.00   0.       0.112    0.340       0.   0.
80700.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
       6 20-86     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9750.000     0.    0.    0.   0
80700.00        0.        0.        0.
3.552   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.5074      0.   0.   0.   0.   0.
0.00   0.       0.097    0.287       0.   0.
80700.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
       7 20-84     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
8800.000     0.    0.    0.   0
81400.00        0.        0.        0.
3.430   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.4900      0.   0.   0.   0.   0.
0.00   0.       0.079    0.230       0.   0.
81400.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068291 0.007788 0.008625 0.009967 0.012202 0.014230 0.015990 0.033335 0.082067
       1         1        0        0     1    1    0    8    18
       8 20-82     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7400.000     0.    0.    0.   0
82000.00        0.        0.        0.
3.280   2.1664   0.0832      0.101   0.   0.   0.
-1.0   0.4686      0.   0.   0.   0.   0.
0.00   0.       0.054    0.158       0.   0.
82000.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
       9 15-88     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
11600.00     0.    0.    0.   0
82700.00        0.        0.        0.
2.843   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.4061      0.   0.   0.   0.   0.
0.00   0.       0.100    0.297       0.   0.
82700.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067869 0.007620 0.008301 0.009356 0.011073 0.012611 0.013937 0.026897 0.063085
       1         1        0        0     1    1    0    8    18
      10 15-86     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
10000.00     0.    0.    0.   0
82700.00        0.        0.        0.
2.745   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.3921      0.   0.   0.   0.   0.
0.00   0.       0.084    0.253       0.   0.
82700.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067869 0.007620 0.008301 0.009356 0.011073 0.012611 0.013937 0.026897 0.063085
       1         1        0        0     1    1    0    8    18
      11 15-84     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9100.000     0.    0.    0.   0
83000.00        0.        0.        0.
2.638   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.3769      0.   0.   0.   0.   0.
0.00   0.       0.068    0.202       0.   0.
83000.00         0       10       0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
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0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
      12 15-82     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7650.000     0.    0.    0.   0
83300.00        0.        0.        0.
2.502   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.3574      0.   0.   0.   0.   0.
0.00   0.       0.046    0.140       0.   0.
83300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067715 0.007559 0.008183 0.009133 0.010660 0.012019 0.013188 0.024545 0.056150
       1         1        0        0     1    1    0    8    18
      13 10-88     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
12100.00     0.    0.    0.   0
83500.00        0.        0.        0.
2.002   2.1666   0.0833      0.101   0.   0.   0.
-1.0   0.2860      0.   0.   0.   0.   0.
0.00   0.       0.084    0.248       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      14 10-86     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
10650.00     0.    0.    0.   0
83500.00        0.        0.        0.
1.920   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.2743      0.   0.   0.   0.   0.
0.00   0.       0.071    0.211       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      15 10-84     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9150.000     0.    0.    0.   0
83400.00        0.        0.        0.
1.830   2.1660   0.0830      0.101   0.   0.   0.
-1.0   0.2614      0.   0.   0.   0.   0.
0.00   0.       0.057    0.167       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      16 10-82     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7650.000     0.    0.    0.   0
83200.00        0.        0.        0.
1.715   2.1666   0.0833      0.101   0.   0.   0.
-1.0   0.2450      0.   0.   0.   0.   0.
0.00   0.       0.039    0.115       0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      17 25-58     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5600.000     0.    0.    0.   0
83000.00        0.        0.        0.
3.405   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.4864      0.   0.   0.   0.   0.
0.00   0.       0.086    0.254       0.   0.
83000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
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      18 25-56     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5600.000     0.    0.    0.   0
83000.00        0.         0.         0.
3.327   2.1020   0.0510      0.101   0.   0.   0.
-1.0   0.4753      0.   0.   0.   0.   0.
0.00   0.       0.074    0.217       0.   0.
83000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
      19 25-54     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5550.000     0.    0.    0.   0
83200.00        0.         0.         0.
3.239   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.4627      0.   0.   0.   0.   0.
0.00   0.       0.060    0.174       0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      20 25-52     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4470.000     0.    0.    0.   0
83400.00        0.         0.         0.
3.117   2.1022   0.0511      0.101   0.   0.   0.
-1.0   0.4453      0.   0.   0.   0.   0.
0.00   0.       0.042    0.123       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      21 20-58     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6300.000     0.    0.    0.   0
83600.00        0.         0.         0.
2.778   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.3969      0.   0.   0.   0.   0.
0.00   0.       0.077    0.232       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      22 20-56     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6300.000     0.    0.    0.   0
83600.00        0.         0.         0.
2.715   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.3879      0.   0.   0.   0.   0.
0.00   0.       0.067    0.194       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      23 20-54     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5450.000     0.    0.    0.   0
83400.00        0.         0.         0.
2.624   2.1022   0.0511      0.101   0.   0.   0.
-1.0   0.3749      0.   0.   0.   0.   0.
0.00   0.       0.053    0.159       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      24 20-52     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4500.000     0.    0.    0.   0
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83300.00        0.        0.        0.
2.522   2.1022   0.0511      0.101   0.   0.   0.
-1.0   0.3603      0.   0.   0.   0.   0.
0.00   0.       0.037    0.110      0.   0.
83300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067715 0.007559 0.008183 0.009133 0.010660 0.012019 0.013188 0.024545 0.056150
       1         1        0        0     1    1    0    8    18
      25 15-58     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7050.000     0.    0.    0.   0
83200.00        0.        0.        0.
2.157   2.1012   0.0506      0.101   0.   0.   0.
-1.0   0.3081      0.   0.   0.   0.   0.
0.00   0.       0.069    0.201      0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      26 15-56     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6400.000     0.    0.    0.   0
83200.00        0.        0.        0.
2.085   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.2979      0.   0.   0.   0.   0.
0.00   0.       0.058    0.174      0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      27 15-54     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5500.000     0.    0.    0.   0
83400.00        0.        0.        0.
2.017   2.1026   0.0513      0.101   0.   0.   0.
-1.0   0.2881      0.   0.   0.   0.   0.
0.00   0.       0.047    0.138      0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      28 15-52     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4600.000     0.    0.    0.   0
83500.00        0.        0.        0.
1.919   2.1028   0.0514      0.101   0.   0.   0.
-1.0   0.2741      0.   0.   0.   0.   0.
0.00   0.       0.032    0.097      0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      29 10-58     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7250.000     0.    0.    0.   0
83700.00        0.        0.        0.
1.506   2.1030   0.0515      0.101   0.   0.   0.
-1.0   0.2151      0.   0.   0.   0.   0.
0.00   0.       0.057    0.169      0.   0.
83700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067623 0.007523 0.008113 0.009002 0.010416 0.011669 0.012744 0.023156 0.052052
       1         1        0        0     1    1    0    8    18
      30 10-56     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6450.000     0.    0.    0.   0
83700.00        0.        0.        0.
1.446   2.1022   0.0511      0.101   0.   0.   0.
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-1.0   0.2066      0.   0.   0.   0.   0.
0.00   0.       0.048    0.146       0.   0.
83700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067623 0.007523 0.008113 0.009002 0.010416 0.011669 0.012744 0.023156 0.052052
       1         1        0        0     1    1    0    8    18
      31 10-54     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5625.000     0.    0.    0.   0
83800.00        0.        0.        0.
1.389   2.1032   0.0516      0.101   0.   0.   0.
-1.0   0.1984      0.   0.   0.   0.   0.
0.00   0.       0.040    0.114       0.   0.
83800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067602 0.007514 0.008097 0.008971 0.010359 0.011587 0.012640 0.022829 0.051088
       1         1        0        0     1    1    0    8    18
      32 10-52     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4700.000     0.    0.    0.   0
84100.00        0.        0.        0.
1.311   2.1036   0.0518      0.101   0.   0.   0.
-1.0   0.1873      0.   0.   0.   0.   0.
0.00   0.       0.027    0.078       0.   0.
84100.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067540 0.007490 0.008050 0.008882 0.010194 0.011351 0.012342 0.021893 0.048328
       1         1        0        0     1    1    0    8    18
      33 25-28     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1850.000     0.    0.    0.   0
78400.00        0.        0.        0.
2.049   2.0400   0.0200      0.101   0.   0.   0.
-1.0   0.2927      0.   0.   0.   0.   0.
0.00   0.       0.043    0.123       0.   0.
78400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005729 0.0069984 0.008461 0.009921 0.012412 0.016726 0.020717 0.024209 0.059116 0.158086
       1         1        0        0     1    1    0    8    18
      34 25-26     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1960.000     0.    0.    0.   0
80600.00        0.        0.        0.
2.008   2.0408   0.0204      0.101   0.   0.   0.
-1.0   0.2869      0.   0.   0.   0.   0.
0.00   0.       0.036    0.106       0.   0.
80600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1         1        0        0     1    1    0    8    18
      35 25-24     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1880.000     0.    0.    0.   0
84300.00        0.        0.        0.
1.965   2.0404   0.0202      0.101   0.   0.   0.
-1.0   0.2807      0.   0.   0.   0.   0.
0.00   0.       0.030    0.086       0.   0.
84300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067502 0.007475 0.008020 0.008826 0.010091 0.011204 0.012154 0.021305 0.046595
       1         1        0        0     1    1    0    8    18
      36 20-28     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2225.000     0.    0.    0.   0
84300.00        0.        0.        0.
1.661   2.0406   0.0203      0.101   0.   0.   0.
-1.0   0.2373      0.   0.   0.   0.   0.
0.00   0.       0.038    0.111       0.   0.
84300.00         0       10        0        0
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60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067502 0.007475 0.008020 0.008826 0.010091 0.011204 0.012154 0.021305 0.046595
       1         1        0        0     1    1    0    8    18
      37 20-26     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2150.000     0.    0.    0.   0
84400.00        0.        0.        0.
1.626   2.0408   0.0204      0.101   0.   0.   0.
-1.0   0.2323      0.   0.   0.   0.   0.
0.00   0.       0.032    0.096       0.   0.
84400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067483 0.007467 0.008006 0.008799 0.010041 0.011132 0.012064 0.021021 0.045758
       1         1        0        0     1    1    0    8    18
      38 20-24     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2075.000     0.    0.    0.   0
84400.00        0.        0.        0.
1.586   2.0420   0.0210      0.101   0.   0.   0.
-1.0   0.2266      0.   0.   0.   0.   0.
0.00   0.       0.026    0.076       0.   0.
84400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067483 0.007467 0.008006 0.008799 0.010041 0.011132 0.012064 0.021021 0.045758
       1         1        0        0     1    1    0    8    18
      39 15-28     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2400.000     0.    0.    0.   0
84000.00        0.        0.        0.
1.280   2.0402   0.0201      0.101   0.   0.   0.
-1.0   0.1829      0.   0.   0.   0.   0.
0.00   0.       0.033    0.099       0.   0.
84000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067560 0.007498 0.008065 0.008911 0.010248 0.011428 0.012439 0.022197 0.049226
       1         1        0        0     1    1    0    8    18
      40 15-26     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2360.000     0.    0.    0.   0
83800.00        0.        0.        0.
1.254   2.0410   0.0205      0.101   0.   0.   0.
-1.0   0.1791      0.   0.   0.   0.   0.
0.00   0.       0.029    0.081       0.   0.
83800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067602 0.007514 0.008097 0.008971 0.010359 0.011587 0.012640 0.022829 0.051088
       1         1        0        0     1    1    0    8    18
      41 15-24     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2060.000     0.    0.    0.   0
83500.00        0.        0.        0.
1.216   2.0412   0.0206      0.101   0.   0.   0.
-1.0   0.1737      0.   0.   0.   0.   0.
0.00   0.       0.023    0.066       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      42 10-28     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2625.000     0.    0.    0.   0
83500.00        0.        0.        0.
0.888   2.0412   0.0206      0.101   0.   0.   0.
-1.0   0.1269      0.   0.   0.   0.   0.
0.00   0.       0.028    0.079       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
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       1         1        0        0     1    1    0    8    18
      43 10-26     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2360.000     0.    0.    0.   0
83500.00        0.        0.         0.
0.863   2.0410   0.0205      0.101   0.   0.   0.
-1.0   0.1233      0.   0.   0.   0.   0.
0.00   0.       0.024    0.068       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      44 10-24     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2040.000     0.    0.    0.   0
83600.00        0.        0.         0.
0.834   2.0406   0.0203      0.101   0.   0.   0.
-1.0   0.1191      0.   0.   0.   0.   0.
0.00   0.       0.019    0.056       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      45 10-22     Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1640.000     0.    0.    0.   0
83600.00        0.        0.         0.
0.793   2.0406   0.0203      0.101   0.   0.   0.
-1.0   0.1133      0.   0.   0.   0.   0.
0.00   0.       0.014    0.039       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      46 15-58F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7400.000     0.    0.    0.   0
82000.00        0.        0.         0.
2.157   2.1022   0.0511      0.101   0.   0.   0.
-1.0   0.3081      0.   0.   0.   0.   0.
0.00   0.       0.067    0.200       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      47 15-56F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6750.000     0.    0.    0.   0
82000.00        0.        0.         0.
2.087   2.1014   0.0507      0.101   0.   0.   0.
-1.0   0.2981      0.   0.   0.   0.   0.
0.00   0.       0.056    0.174       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      48 15-54F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5850.000     0.    0.    0.   0
82000.00        0.        0.         0.
2.018   2.1024   0.0512      0.101   0.   0.   0.
-1.0   0.2883      0.   0.   0.   0.   0.
0.00   0.       0.042    0.134       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      49 15-52F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
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4920.000     0.    0.    0.   0
82000.00        0.        0.        0.
1.921   2.1020   0.0510      0.101   0.   0.   0.
-1.0   0.2744      0.   0.   0.   0.   0.
0.00   0.       0.030    0.097       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      50 10-58F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7700.000     0.    0.    0.   0
82000.00        0.        0.        0.
1.507   2.1030   0.0515      0.101   0.   0.   0.
-1.0   0.2153      0.   0.   0.   0.   0.
0.00   0.       0.055    0.169       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      51 10-56F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6800.000     0.    0.    0.   0
82500.00        0.        0.        0.
1.446   2.1034   0.0517      0.101   0.   0.   0.
-1.0   0.2066      0.   0.   0.   0.   0.
0.00   0.       0.047    0.145       0.   0.
82500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067926 0.007643 0.008345 0.009438 0.011224 0.012828 0.014212 0.027760 0.065629
       1         1        0        0     1    1    0    8    18
      52 10-54F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5950.000     0.    0.    0.   0
82900.00        0.        0.        0.
1.390   2.1020   0.0510      0.101   0.   0.   0.
-1.0   0.1986      0.   0.   0.   0.   0.
0.00   0.       0.036    0.109       0.   0.
82900.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067815 0.007599 0.008260 0.009279 0.010928 0.012404 0.013675 0.026075 0.060661
       1         1        0        0     1    1    0    8    18
      53 10-52F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5000.000     0.    0.    0.   0
83400.00        0.        0.        0.
1.315   2.1018   0.0509      0.101   0.   0.   0.
-1.0   0.1879      0.   0.   0.   0.   0.
0.00   0.       0.025    0.079       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      54 25-28F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2160.000     0.    0.    0.   0
83400.00        0.        0.        0.
2.048   2.0418   0.0209      0.101   0.   0.   0.
-1.0   0.2926      0.   0.   0.   0.   0.
0.00   0.       0.040    0.122       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      55 25-26F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2080.000     0.    0.    0.   0
82800.00        0.        0.        0.
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2.010   2.0418   0.0209      0.101   0.   0.   0.
-1.0   0.2871      0.   0.   0.   0.   0.
0.00   0.       0.030    0.105       0.   0.
82800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067842 0.007610 0.008281 0.009317 0.011000 0.012506 0.013805 0.026481 0.061859
       1         1        0        0     1    1    0    8    18
      56 25-24F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1935.000     0.    0.    0.   0
82300.00        0.        0.         0.
1.967   2.0400   0.0200      0.101   0.   0.   0.
-1.0   0.2810      0.   0.   0.   0.   0.
0.00   0.       0.029    0.086       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      57 25-22F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1420.000     0.    0.    0.   0
81700.00        0.        0.         0.
1.910   2.0410   0.0205      0.101   0.   0.   0.
-1.0   0.2729      0.   0.   0.   0.   0.
0.00   0.       0.019    0.060       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      58 20-28F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2135.000     0.    0.    0.   0
81700.00        0.        0.         0.
1.665   2.0386   0.0193      0.101   0.   0.   0.
-1.0   0.2379      0.   0.   0.   0.   0.
0.00   0.       0.036    0.112       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      59 20-26F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2100.000     0.    0.    0.   0
81300.00        0.        0.         0.
1.626   2.0386   0.0193      0.101   0.   0.   0.
-1.0   0.2323      0.   0.   0.   0.   0.
0.00   0.       0.030    0.097       0.   0.
81300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068330 0.007804 0.008654 0.010022 0.012305 0.014378 0.016176 0.033920 0.083793
       1         1        0        0     1    1    0    8    18
      60 20-24F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2060.000     0.    0.    0.   0
80800.00        0.        0.         0.
1.588   2.0398   0.0199      0.101   0.   0.   0.
-1.0   0.2269      0.   0.   0.   0.   0.
0.00   0.       0.026    0.077       0.   0.
80800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005722 0.0068537 0.007886 0.008813 0.010321 0.012858 0.015171 0.017181 0.037072 0.093088
       1         1        0        0     1    1    0    8    18
      61 20-22F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1520.000     0.    0.    0.   0
80400.00        0.        0.         0.
1.538   2.0394   0.0197      0.101   0.   0.   0.
-1.0   0.2197      0.   0.   0.   0.   0.
0.00   0.       0.018    0.054       0.   0.
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80400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068722 0.007959 0.008954 0.010589 0.013353 0.015880 0.018080 0.039891 0.101398
       1         1        0        0     1    1    0    8    18
      62 15-28F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2580.000     0.    0.    0.   0
80400.00        0.        0.        0.
1.278   2.0414   0.0207      0.101   0.   0.   0.
-1.0   0.1826      0.   0.   0.   0.   0.
0.00   0.       0.033    0.097       0.   0.
80400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068722 0.007959 0.008954 0.010589 0.013353 0.015880 0.018080 0.039891 0.101398
       1         1        0        0     1    1    0    8    18
      63 15-26F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2400.000     0.    0.    0.   0
81100.00        0.        0.        0.
1.255   2.0408   0.0204      0.101   0.   0.   0.
-1.0   0.1793      0.   0.   0.   0.   0.
0.00   0.       0.028    0.081       0.   0.
81100.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005722 0.0068410 0.007835 0.008715 0.010138 0.012518 0.014683 0.016563 0.035135 0.087374
       1         1        0        0     1    1    0    8    18
      64 15-24F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2180.000     0.    0.    0.   0
81700.00        0.        0.        0.
1.217   2.0416   0.0208      0.101   0.   0.   0.
-1.0   0.1739      0.   0.   0.   0.   0.
0.00   0.       0.023    0.067       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      65 15-22F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1720.000     0.    0.    0.   0
82400.00        0.        0.        0.
1.173   2.0406   0.0203      0.101   0.   0.   0.
-1.0   0.1676      0.   0.   0.   0.   0.
0.00   0.       0.016    0.049       0.   0.
82400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067955 0.007655 0.008367 0.009481 0.011302 0.012940 0.014355 0.028208 0.066949
       1         1        0        0     1    1    0    8    18
      66 10-28F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2840.000     0.    0.    0.   0
82400.00        0.        0.        0.
0.889   2.0426   0.0213      0.101   0.   0.   0.
-1.0   0.1270      0.   0.   0.   0.   0.
0.00   0.       0.028    0.079       0.   0.
82400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067955 0.007655 0.008367 0.009481 0.011302 0.012940 0.014355 0.028208 0.066949
       1         1        0        0     1    1    0    8    18
      67 10-26F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2540.000     0.    0.    0.   0
82300.00        0.        0.        0.
0.861   2.0404   0.0202      0.101   0.   0.   0.
-1.0   0.1230      0.   0.   0.   0.   0.
0.00   0.       0.027    0.068       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
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0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      68 10-24F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2180.000     0.    0.    0.   0
82300.00        0.        0.        0.
0.832   2.0398   0.0199      0.101   0.   0.   0.
-1.0   0.1189      0.   0.   0.   0.   0.
0.00   0.       0.019    0.057       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      69 10-22F    Average Lf, actual shell length. Compare w/ general.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1775.000     0.    0.    0.   0
82200.00        0.        0.        0.
0.795   2.0400   0.0200      0.101   0.   0.   0.
-1.0   0.1136      0.   0.   0.   0.   0.
0.00   0.       0.015    0.039       0.   0.
82200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068016 0.007679 0.008414 0.009569 0.011465 0.013174 0.014651 0.029135 0.069685
```

The following is the DAPS4 input deck for computing the interbay buckling pressure, axisymmetric collapse pressure, stresses and deflections for the Boichot and Reynolds tests of reference A-1. For this set of calculations, the actual ring spacing, L_f, is input along with a longer shell length (L_f x the number of bays) to best represent the local ring-shell stiffness for calculation of these modes of collapse and the stresses and deflections from mid-bay to the ring. Since the input value for L is not the actual, the computed general instability pressure is ignored for this set of calculations.

```
       1         1        0        0     1    1    0    8    18
       1 25-88     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9450.000     0.    0.    0.   0
80600.00        0.        0.        0.
4.8930    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.699       0.   0.   0.   0.   0.
0.00   0.       0.127    0.386       0.   0.
80600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1         1        0        0     1    1    0    8    18
       2 25-86     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9550.000     0.    0.    0.   0
80600.00        0.        0.        0.
4.7530    2.1664    0.0832       0.101   0.   0.   0.
-1.0    0.679       0.   0.   0.   0.   0.
0.00   0.       0.107    0.318       0.   0.
80600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1         1        0        0     1    1    0    8    18
       3 25-84     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
8850.000     0.    0.    0.   0
80700.00        0.        0.        0.
4.6130    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.659       0.   0.   0.   0.   0.
0.00   0.       0.086    0.255       0.   0.
80700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
```

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       4 25-82     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7275.000     0.    0.    0.   0
80700.00        0.         0.         0.
4.4240    2.1652    0.0826       0.101   0.   0.   0.
-1.0    0.632       0.   0.   0.   0.   0.
0.00   0.       0.059    0.179       0.   0.
80700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
       5 20-88     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
10600.00     0.    0.    0.   0
80700.00        0.         0.         0.
4.0040    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.572       0.   0.   0.   0.   0.
0.00   0.       0.112    0.340       0.   0.
80700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
       6 20-86     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9750.000     0.    0.    0.   0
80700.00        0.         0.         0.
3.8850    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.555       0.   0.   0.   0.   0.
0.00   0.       0.097    0.287       0.   0.
80700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068581 0.007903 0.008847 0.010386 0.012977 0.015342 0.017398 0.037751 0.095089
       1         1        0        0     1    1    0    8    18
       7 20-84     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
8800.000     0.    0.    0.   0
81400.00        0.         0.         0.
3.7590    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.537       0.   0.   0.   0.   0.
0.00   0.       0.079    0.230       0.   0.
81400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068291 0.007788 0.008625 0.009967 0.012202 0.014230 0.015990 0.033335 0.082067
       1         1        0        0     1    1    0    8    18
       8 20-82     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7400.000     0.    0.    0.   0
82000.00        0.         0.         0.
3.5770    2.1664    0.0832       0.101   0.   0.   0.
-1.0    0.511       0.   0.   0.   0.   0.
0.00   0.       0.054    0.158       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
       9 15-88     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
11600.00     0.    0.    0.   0
82700.00        0.         0.         0.
3.1080    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.444       0.   0.   0.   0.   0.
0.00   0.       0.100    0.297       0.   0.
82700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067869 0.007620 0.008301 0.009356 0.011073 0.012611 0.013937 0.026897 0.063085
       1         1        0        0     1    1    0    8    18
      10 15-86     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
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10000.00     0.    0.    0.   0
82700.00        0.        0.        0.
2.9960    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.428       0.   0.   0.   0.   0.
0.00   0.       0.084    0.253       0.   0.
82700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067869 0.007620 0.008301 0.009356 0.011073 0.012611 0.013937 0.026897 0.063085
       1         1        0        0     1    1    0    8    18
      11 15-84     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9100.000     0.    0.    0.   0
83000.00        0.        0.        0.
2.8910    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.413       0.   0.   0.   0.   0.
0.00   0.       0.068    0.202       0.   0.
83000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
      12 15-82     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7650.000     0.    0.    0.   0
83300.00        0.        0.        0.
2.7300    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.390       0.   0.   0.   0.   0.
0.00   0.       0.046    0.140       0.   0.
83300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067715 0.007559 0.008183 0.009133 0.010660 0.012019 0.013188 0.024545 0.056150
       1         1        0        0     1    1    0    8    18
      13 10-88     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
12100.00     0.    0.    0.   0
83500.00        0.        0.        0.
2.1910    2.1666    0.0833       0.101   0.   0.   0.
-1.0    0.313       0.   0.   0.   0.   0.
0.00   0.       0.084    0.248       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      14 10-86     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
10650.00     0.    0.    0.   0
83500.00        0.        0.        0.
2.1000    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.300       0.   0.   0.   0.   0.
0.00   0.       0.071    0.211       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      15 10-84     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
9150.000     0.    0.    0.   0
83400.00        0.        0.        0.
2.0020    2.1660    0.0830       0.101   0.   0.   0.
-1.0    0.286       0.   0.   0.   0.   0.
0.00   0.       0.057    0.167       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      16 10-82     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7650.000     0.    0.    0.   0
83200.00        0.        0.        0.
1.8830    2.1666    0.0833       0.101   0.   0.   0.
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-1.0    0.269       0.   0.   0.   0.   0.
0.00   0.       0.039    0.115       0.   0.
83200.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      17 25-58     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5600.000     0.    0.    0.   0
83000.00        0.        0.         0.
3.7240    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.532       0.   0.   0.   0.   0.
0.00   0.       0.086    0.254       0.   0.
83000.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
      18 25-56     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5600.000     0.    0.    0.   0
83000.00        0.        0.         0.
3.6400    2.1020    0.0510       0.101   0.   0.   0.
-1.0    0.520       0.   0.   0.   0.   0.
0.00   0.       0.074    0.217       0.   0.
83000.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067789 0.007589 0.008240 0.009241 0.010859 0.012304 0.013549 0.025679 0.059493
       1         1        0        0     1    1    0    8    18
      19 25-54     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5550.000     0.    0.    0.   0
83200.00        0.        0.         0.
3.5420    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.506       0.   0.   0.   0.   0.
0.00   0.       0.060    0.174       0.   0.
83200.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      20 25-52     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4470.000     0.    0.    0.   0
83400.00        0.        0.         0.
3.4090    2.1022    0.0511       0.101   0.   0.   0.
-1.0    0.487       0.   0.   0.   0.   0.
0.00   0.       0.042    0.123       0.   0.
83400.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      21 20-58     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6300.000     0.    0.    0.   0
83600.00        0.        0.         0.
3.0380    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.434       0.   0.   0.   0.   0.
0.00   0.       0.077    0.232       0.   0.
83600.00         0       10         0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      22 20-56     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6300.000     0.    0.    0.   0
83600.00        0.        0.         0.
2.9680    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.424       0.   0.   0.   0.   0.
0.00   0.       0.067    0.194       0.   0.
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83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      23 20-54     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5450.000     0.    0.    0.   0
83400.00        0.        0.        0.
2.8700    2.1022    0.0511       0.101   0.   0.   0.
-1.0    0.410       0.   0.   0.   0.   0.
0.00   0.       0.053    0.159       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      24 20-52     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4500.000     0.    0.    0.   0
83300.00        0.        0.        0.
2.7580    2.1022    0.0511       0.101   0.   0.   0.
-1.0    0.394       0.   0.   0.   0.   0.
0.00   0.       0.037    0.110       0.   0.
83300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067715 0.007559 0.008183 0.009133 0.010660 0.012019 0.013188 0.024545 0.056150
       1         1        0        0     1    1    0    8    18
      25 15-58     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7050.000     0.    0.    0.   0
83200.00        0.        0.        0.
2.3590    2.1012    0.0506       0.101   0.   0.   0.
-1.0    0.337       0.   0.   0.   0.   0.
0.00   0.       0.069    0.201       0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      26 15-56     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6400.000     0.    0.    0.   0
83200.00        0.        0.        0.
2.2820    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.326       0.   0.   0.   0.   0.
0.00   0.       0.058    0.174       0.   0.
83200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067739 0.007569 0.008202 0.009168 0.010725 0.012112 0.013305 0.024914 0.057238
       1         1        0        0     1    1    0    8    18
      27 15-54     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5500.000     0.    0.    0.   0
83400.00        0.        0.        0.
2.2050    2.1026    0.0513       0.101   0.   0.   0.
-1.0    0.315       0.   0.   0.   0.   0.
0.00   0.       0.047    0.138       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      28 15-52     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4600.000     0.    0.    0.   0
83500.00        0.        0.        0.
2.1000    2.1028    0.0514       0.101   0.   0.   0.
-1.0    0.300       0.   0.   0.   0.   0.
0.00   0.       0.032    0.097       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
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       1         1        0        0     1    1    0    8    18
      29 10-58     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7250.000     0.    0.    0.   0
83700.00       0.        0.        0.
1.6450    2.1030    0.0515       0.101   0.   0.   0.
-1.0    0.235       0.   0.   0.   0.   0.
0.00   0.       0.057    0.169       0.   0.
83700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067623 0.007523 0.008113 0.009002 0.010416 0.011669 0.012744 0.023156 0.052052
       1         1        0        0     1    1    0    8    18
      30 10-56     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6450.000     0.    0.    0.   0
83700.00       0.        0.        0.
1.5820    2.1022    0.0511       0.101   0.   0.   0.
-1.0    0.226       0.   0.   0.   0.   0.
0.00   0.       0.048    0.146       0.   0.
83700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067623 0.007523 0.008113 0.009002 0.010416 0.011669 0.012744 0.023156 0.052052
       1         1        0        0     1    1    0    8    18
      31 10-54     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5625.000     0.    0.    0.   0
83800.00       0.        0.        0.
1.5190    2.1032    0.0516       0.101   0.   0.   0.
-1.0    0.217       0.   0.   0.   0.   0.
0.00   0.       0.040    0.114       0.   0.
83800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067602 0.007514 0.008097 0.008971 0.010359 0.011587 0.012640 0.022829 0.051088
       1         1        0        0     1    1    0    8    18
      32 10-52     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4700.000     0.    0.    0.   0
84100.00       0.        0.        0.
1.4350    2.1036    0.0518       0.101   0.   0.   0.
-1.0    0.205       0.   0.   0.   0.   0.
0.00   0.       0.027    0.078       0.   0.
84100.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067540 0.007490 0.008050 0.008882 0.010194 0.011351 0.012342 0.021893 0.048328
       1         1        0        0     1    1    0    8    18
      33 25-28     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1850.000     0.    0.    0.   0
78400.00       0.        0.        0.
2.2400    2.0400    0.0200       0.101   0.   0.   0.
-1.0    0.320       0.   0.   0.   0.   0.
0.00   0.       0.043    0.123       0.   0.
78400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005729 0.0069984 0.008461 0.009921 0.012412 0.016726 0.020717 0.024209 0.059116 0.158086
       1         1        0        0     1    1    0    8    18
      34 25-26     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1960.000     0.    0.    0.   0
80600.00       0.        0.        0.
2.1980    2.0408    0.0204       0.101   0.   0.   0.
-1.0    0.314       0.   0.   0.   0.   0.
0.00   0.       0.036    0.106       0.   0.
80600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068627 0.007922 0.008882 0.010452 0.013099 0.015517 0.017619 0.038447 0.097140
       1         1        0        0     1    1    0    8    18
      35 25-24     Actual Lf, longer shell.  Compare interbay/stresses.
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1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1880.000     0.    0.    0.   0
84300.00        0.        0.        0.
2.1490    2.0404    0.0202       0.101   0.   0.   0.
-1.0    0.307       0.   0.   0.   0.   0.
0.00   0.       0.030    0.086       0.   0.
84300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067502 0.007475 0.008020 0.008826 0.010091 0.011204 0.012154 0.021305 0.046595
       1         1        0        0     1    1    0    8    18
      36 20-28     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2225.000     0.    0.    0.   0
84300.00        0.        0.        0.
1.8200    2.0406    0.0203       0.101   0.   0.   0.
-1.0    0.260       0.   0.   0.   0.   0.
0.00   0.       0.038    0.111       0.   0.
84300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067502 0.007475 0.008020 0.008826 0.010091 0.011204 0.012154 0.021305 0.046595
       1         1        0        0     1    1    0    8    18
      37 20-26     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2150.000     0.    0.    0.   0
84400.00        0.        0.        0.
1.7710    2.0408    0.0204       0.101   0.   0.   0.
-1.0    0.253       0.   0.   0.   0.   0.
0.00   0.       0.032    0.096       0.   0.
84400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067483 0.007467 0.008006 0.008799 0.010041 0.011132 0.012064 0.021021 0.045758
       1         1        0        0     1    1    0    8    18
      38 20-24     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2075.000     0.    0.    0.   0
84400.00        0.        0.        0.
1.7360    2.0420    0.0210       0.101   0.   0.   0.
-1.0    0.248       0.   0.   0.   0.   0.
0.00   0.       0.026    0.076       0.   0.
84400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067483 0.007467 0.008006 0.008799 0.010041 0.011132 0.012064 0.021021 0.045758
       1         1        0        0     1    1    0    8    18
      39 15-28     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2400.000     0.    0.    0.   0
84000.00        0.        0.        0.
1.4000    2.0402    0.0201       0.101   0.   0.   0.
-1.0    0.200       0.   0.   0.   0.   0.
0.00   0.       0.033    0.099       0.   0.
84000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067560 0.007498 0.008065 0.008911 0.010248 0.011428 0.012439 0.022197 0.049226
       1         1        0        0     1    1    0    8    18
      40 15-26     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2360.000     0.    0.    0.   0
83800.00        0.        0.        0.
1.3720    2.0410    0.0205       0.101   0.   0.   0.
-1.0    0.196       0.   0.   0.   0.   0.
0.00   0.       0.029    0.081       0.   0.
83800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067602 0.007514 0.008097 0.008971 0.010359 0.011587 0.012640 0.022829 0.051088
       1         1        0        0     1    1    0    8    18
      41 15-24     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2060.000     0.    0.    0.   0
83500.00        0.        0.        0.
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1.3300    2.0412    0.0206       0.101   0.   0.   0.
-1.0    0.190       0.   0.   0.   0.   0.
0.00   0.       0.023    0.066       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      42 10-28     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2625.000     0.    0.    0.   0
83500.00        0.        0.        0.
0.9800    2.0412    0.0206       0.101   0.   0.   0.
-1.0    0.140       0.   0.   0.   0.   0.
0.00   0.       0.028    0.079       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      43 10-26     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2360.000     0.    0.    0.   0
83500.00        0.        0.        0.
0.9450    2.0410    0.0205       0.101   0.   0.   0.
-1.0    0.135       0.   0.   0.   0.   0.
0.00   0.       0.024    0.068       0.   0.
83500.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067668 0.007540 0.008148 0.009066 0.010535 0.011840 0.012961 0.023834 0.054052
       1         1        0        0     1    1    0    8    18
      44 10-24     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2040.000     0.    0.    0.   0
83600.00        0.        0.        0.
0.9100    2.0406    0.0203       0.101   0.   0.   0.
-1.0    0.130       0.   0.   0.   0.   0.
0.00   0.       0.019    0.056       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      45 10-22     Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1640.000     0.    0.    0.   0
83600.00        0.        0.        0.
0.8680    2.0406    0.0203       0.101   0.   0.   0.
-1.0    0.124       0.   0.   0.   0.   0.
0.00   0.       0.014    0.039       0.   0.
83600.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067645 0.007532 0.008130 0.009033 0.010475 0.011754 0.012851 0.023491 0.053040
       1         1        0        0     1    1    0    8    18
      46 15-58F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7400.000     0.    0.    0.   0
82000.00        0.        0.        0.
2.3590    2.1022    0.0511       0.101   0.   0.   0.
-1.0    0.337       0.   0.   0.   0.   0.
0.00   0.       0.067    0.200       0.   0.
82000.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      47 15-56F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6750.000     0.    0.    0.   0
82000.00        0.        0.        0.
2.2750    2.1014    0.0507       0.101   0.   0.   0.
-1.0    0.325       0.   0.   0.   0.   0.
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0.00   0.       0.056    0.174      0.   0.
82000.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      48 15-54F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5850.000     0.    0.    0.   0
82000.00        0.        0.        0.
2.1910    2.1024    0.0512      0.101   0.   0.   0.
-1.0    0.313       0.   0.   0.   0.   0.
0.00   0.       0.042    0.134      0.   0.
82000.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      49 15-52F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
4920.000     0.    0.    0.   0
82000.00        0.        0.        0.
2.1000    2.1020    0.0510      0.101   0.   0.   0.
-1.0    0.300       0.   0.   0.   0.   0.
0.00   0.       0.030    0.097      0.   0.
82000.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      50 10-58F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
7700.000     0.    0.    0.   0
82000.00        0.        0.        0.
1.6450    2.1030    0.0515      0.101   0.   0.   0.
-1.0    0.235       0.   0.   0.   0.   0.
0.00   0.       0.055    0.169      0.   0.
82000.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068080 0.007704 0.008463 0.009661 0.011636 0.013419 0.014961 0.030110 0.072557
       1         1        0        0     1    1    0    8    18
      51 10-56F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
6800.000     0.    0.    0.   0
82500.00        0.        0.        0.
1.5750    2.1034    0.0517      0.101   0.   0.   0.
-1.0    0.225       0.   0.   0.   0.   0.
0.00   0.       0.047    0.145      0.   0.
82500.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067926 0.007643 0.008345 0.009438 0.011224 0.012828 0.014212 0.027760 0.065629
       1         1        0        0     1    1    0    8    18
      52 10-54F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5950.000     0.    0.    0.   0
82900.00        0.        0.        0.
1.5120    2.1020    0.0510      0.101   0.   0.   0.
-1.0    0.216       0.   0.   0.   0.   0.
0.00   0.       0.036    0.109      0.   0.
82900.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067815 0.007599 0.008260 0.009279 0.010928 0.012404 0.013675 0.026075 0.060661
       1         1        0        0     1    1    0    8    18
      53 10-52F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
5000.000     0.    0.    0.   0
83400.00        0.        0.        0.
1.4350    2.1018    0.0509      0.101   0.   0.   0.
-1.0    0.205       0.   0.   0.   0.   0.
0.00   0.       0.025    0.079      0.   0.
83400.00        0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
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0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      54 25-28F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2160.000     0.    0.    0.   0
83400.00        0.         0.         0.
2.2400    2.0418    0.0209       0.101   0.   0.   0.
-1.0    0.320       0.   0.   0.   0.   0.
0.00   0.       0.040    0.122       0.   0.
83400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005718 0.0067691 0.007550 0.008165 0.009099 0.010597 0.011928 0.013073 0.024185 0.055089
       1         1        0        0     1    1    0    8    18
      55 25-26F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2080.000     0.    0.    0.   0
82800.00        0.         0.         0.
2.1700    2.0418    0.0209       0.101   0.   0.   0.
-1.0    0.310       0.   0.   0.   0.   0.
0.00   0.       0.030    0.105       0.   0.
82800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005719 0.0067842 0.007610 0.008281 0.009317 0.011000 0.012506 0.013805 0.026481 0.061859
       1         1        0        0     1    1    0    8    18
      56 25-24F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1935.000     0.    0.    0.   0
82300.00        0.         0.         0.
2.1490    2.0400    0.0200       0.101   0.   0.   0.
-1.0    0.307       0.   0.   0.   0.   0.
0.00   0.       0.029    0.086       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      57 25-22F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1420.000     0.    0.    0.   0
81700.00        0.         0.         0.
2.0860    2.0410    0.0205       0.101   0.   0.   0.
-1.0    0.298       0.   0.   0.   0.   0.
0.00   0.       0.019    0.060       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      58 20-28F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2135.000     0.    0.    0.   0
81700.00        0.         0.         0.
1.8200    2.0386    0.0193       0.101   0.   0.   0.
-1.0    0.260       0.   0.   0.   0.   0.
0.00   0.       0.036    0.112       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      59 20-26F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2100.000     0.    0.    0.   0
81300.00        0.         0.         0.
1.7710    2.0386    0.0193       0.101   0.   0.   0.
-1.0    0.253       0.   0.   0.   0.   0.
0.00   0.       0.030    0.097       0.   0.
81300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068330 0.007804 0.008654 0.010022 0.012305 0.014378 0.016176 0.033920 0.083793
       1         1        0        0     1    1    0    8    18
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      60 20-24F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2060.000     0.    0.    0.   0
80800.00        0.         0.         0.
1.7290    2.0398    0.0199       0.101   0.   0.   0.
-1.0    0.247       0.   0.   0.   0.   0.
0.00   0.       0.026    0.077       0.   0.
80800.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005722 0.0068537 0.007886 0.008813 0.010321 0.012858 0.015171 0.017181 0.037072 0.093088
       1         1        0        0     1    1    0    8    18
      61 20-22F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1520.000     0.    0.    0.   0
80400.00        0.         0.         0.
1.6870    2.0394    0.0197       0.101   0.   0.   0.
-1.0    0.241       0.   0.   0.   0.   0.
0.00   0.       0.018    0.054       0.   0.
80400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068722 0.007959 0.008954 0.010589 0.013353 0.015880 0.018080 0.039891 0.101398
       1         1        0        0     1    1    0    8    18
      62 15-28F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2580.000     0.    0.    0.   0
80400.00        0.         0.         0.
1.3930    2.0414    0.0207       0.101   0.   0.   0.
-1.0    0.199       0.   0.   0.   0.   0.
0.00   0.       0.033    0.097       0.   0.
80400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005723 0.0068722 0.007959 0.008954 0.010589 0.013353 0.015880 0.018080 0.039891 0.101398
       1         1        0        0     1    1    0    8    18
      63 15-26F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2400.000     0.    0.    0.   0
81100.00        0.         0.         0.
1.3720    2.0408    0.0204       0.101   0.   0.   0.
-1.0    0.196       0.   0.   0.   0.   0.
0.00   0.       0.028    0.081       0.   0.
81100.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005722 0.0068410 0.007835 0.008715 0.010138 0.012518 0.014683 0.016563 0.035135 0.087374
       1         1        0        0     1    1    0    8    18
      64 15-24F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2180.000     0.    0.    0.   0
81700.00        0.         0.         0.
1.3300    2.0416    0.0208       0.101   0.   0.   0.
-1.0    0.190       0.   0.   0.   0.   0.
0.00   0.       0.023    0.067       0.   0.
81700.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005721 0.0068182 0.007745 0.008541 0.009808 0.011909 0.013810 0.015457 0.031663 0.077138
       1         1        0        0     1    1    0    8    18
      65 15-22F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1720.000     0.    0.    0.   0
82400.00        0.         0.         0.
1.2810    2.0406    0.0203       0.101   0.   0.   0.
-1.0    0.183       0.   0.   0.   0.   0.
0.00   0.       0.016    0.049       0.   0.
82400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067955 0.007655 0.008367 0.009481 0.011302 0.012940 0.014355 0.028208 0.066949
       1         1        0        0     1    1    0    8    18
      66 10-28F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2840.000     0.    0.    0.   0
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82400.00        0.        0.        0.
0.9730    2.0426    0.0213       0.101   0.   0.   0.
-1.0    0.139       0.   0.   0.   0.   0.
0.00   0.       0.028    0.079       0.   0.
82400.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067955 0.007655 0.008367 0.009481 0.011302 0.012940 0.014355 0.028208 0.066949
       1         1        0        0     1    1    0    8    18
      67 10-26F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2540.000     0.    0.    0.   0
82300.00        0.        0.        0.
0.9590    2.0404    0.0202       0.101   0.   0.   0.
-1.0    0.137       0.   0.   0.   0.   0.
0.00   0.       0.027    0.068       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      68 10-24F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
2180.000     0.    0.    0.   0
82300.00        0.        0.        0.
0.9100    2.0398    0.0199       0.101   0.   0.   0.
-1.0    0.130       0.   0.   0.   0.   0.
0.00   0.       0.019    0.057       0.   0.
82300.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0067985 0.007667 0.008390 0.009524 0.011383 0.013056 0.014501 0.028666 0.068300
       1         1        0        0     1    1    0    8    18
      69 10-22F    Actual Lf, longer shell.  Compare interbay/stresses.
1.05E+07   0.   0.   0.   0.       0.3     0.35   0.
1775.000     0.    0.    0.   0
82200.00        0.        0.        0.
0.8750    2.0400    0.0200       0.101   0.   0.   0.
-1.0    0.125       0.   0.   0.   0.   0.
0.00   0.       0.015    0.039       0.   0.
82200.00         0       10        0        0
60000.00 70000.000 75000.00 77500.00 80000.00 82500.00 84000.00 85000.00 90000.00 95000.00
0.005720 0.0068016 0.007679 0.008414 0.009569 0.011465 0.013174 0.014651 0.029135 0.069685
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References

A-1. Boichot, Lance and Reynolds, Thomas E., “Inelastic Buckling Tests of Ring-Stiffened Cylinders under Hydrostatic Pressure,” David Taylor Model Basin, DTMB Report 1992, May 1965.

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